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T- and *CP*-symmetric quantization of fields without zero-point energy

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Abstract

The standard replacement of negatively-frequency operators of relativistic fields to operators of antiparticles leads to incomplete time-reversal in field's observables. In fact, a correct transition to the operators of antiparticles occurs only at a full *T*- or *CP*-transformation of matrix elements. Then the operators of observables of *T* - or *CP*-symmetric fields with standard Lagrangians appear as normal-ordered and the zero-point energy and the zero-point charge do not arise. The rules for the *CP*-symmetric quantization of fields are formulated and also the theorem about vanishing of the zero-point energy is proved. The theorem is in agreement with all known experiments since in fact the contributions of real sources are observed only. The divergent zero-point energy and zero-point charge appear only at using of the Lagrangians symmetrized under the complex-conjugate field operators, while the standard Lagrangians without such symmetrization lead to the required vanishing vacuum energy. At spontaneous symmetry breaking the vacuum zero-point energy and zero-point charge do not appear only if the remained scalar field is complex one and in this case the Standard Model predicts a pair boson-antiboson with hypercharge.

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Content

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Introduction

Quantum field theory's (QFT) prediction of zero-point vacuum fluctuations is one of most contradictory problems of particle physics. At the same time, a clear solution of this problem is very important for understanding of physical foundations of QFT and its applications to particle physics and cosmology.

The present paradigm, widely accepted mainly due to historical reasons [1-5], can be reduced to following statements:

a) The zero-point fluctuations of free fields exist, but corresponding zero-point energies are divergent;

b) The zero-point vacuum energy *is unobservable* (with except an ambiguity with its gravitational effect) and the vacuum energy can be *shifted to zero*;

c) The effects caused by differences in spectra of zero-point fluctuations of fields in some cases are *observable* and *already they are measured* in the experiments (the Lamb shift, the Casimir effect).

However, as it will be discussed in section 3.3, in all known cases, where the zero-point fluctuations of fields should be measured in the experiments, the really observed effects can be described completely in terms of the known interactions of real sources without any reference to the vacuum fields [7-10]. Therefore, from these experiments, contrary to the conventional opinions, follow that in fact the zero-point fluctuations of fields have not found out and, moreover, that their contributions are excluded by the existing experiments with very high accuracy [10].

The zero-point energy has been derived in the conventional treatments of QFT as an artifact at the transition from the operator products of negative-frequency particles to the operator products of positive-frequency ones. At early period of QFT some methods of the eliminating of the zero-point energy by means of negative-frequency modes have been considered [11-13]. Later various models without the vacuum energy have been studied, but due to using of additional artificial assumptions, such as negative probabilities and others, changing of the standard principles of quantum mechanics, they have not led to the experimentally checked solutions of the problem. The zero-point energy is absent in a supersymmetric world also, but, as it is known, this property does not solve the problem in the real world.

In the present paper the standard methods of quantization are improved so that the *replacement of the negative-frequency operators* of relativistic fields to the positive-frequency ones is performed not in the field operators as usually, but, due to the nonlinearity of the time-reversal operation, *in the operator products* only. It will be shown that for this purpose it is enough to use the *CP*-symmetry properties of observables giving required identities between different frequency-sign operator products. As the result, for the standard Lagrangians of QFT the zero-point vacuum energy and zero-point vacuum charge do not arise. A direct analogy by the spectrum of non-relativistic harmonic oscillator, where there is the zero-point energy [14], holds only for the *symmetrized* Lagrangians considered in Appendix 2, leading to divergent zero-point vacuum fluctuations and, therefore, such Lagrangians are excluded by the existing experiments.

1. *C*-symmetric quantization of complex fields

1.1 The field operators, states and observables

Let us consider a charged spinless particle of mass m described by the complex scalar field φ. This complex field is equivalent to the system of two real scalar fields φ_1 and φ_2 of the same mass m with the free Lagrangian:

$$L(x)=\frac{1}{2}\Big[(\partial_\mu\varphi_1)(\partial^\mu\varphi_1)+(\partial_\mu\varphi_2)(\partial^\mu\varphi_2)-m^2\left(\varphi_1^2+\varphi_2^2\right)\Big]. \tag{1}$$

After the substitution $\varphi=\varphi_1+i\varphi_2$ we obtain a symmetrized Lagrangian:

$$L_s(x)=\frac{1}{2}\Big[\left(\partial_\mu\varphi^*\right)\left(\partial^\mu\varphi\right)+\left(\partial_\mu\varphi\right)\left(\partial^\mu\varphi^*\right)-m^2\left(\varphi\varphi^*+\varphi^*\varphi\right)\Big]. \tag{2}$$

In this Lagrangian the fields φ, φ^* and their derivatives enter in a symmetrized form and, therefore, the Lagrangian is invariant under the charge conjugation and time reversal symmetries. Moreover, it is a field analog of the Lagrangian of a chain of bounded oscillators and, because of the symmetry between the canonically conjugate variables, at quantization corresponding Hamiltonian will contain a zero-point energy.

As it has been noted in [14], in QFT a direct analogy between the harmonic oscillator's spectrum and the field modes exists only for the Hamiltonians of fields symmetrized under the canonically-conjugate variables. It is related by the fact that the positive- and negative-frequency contributions concern to different subspaces of the total Fock space and, consequently, the values of the observables, having a direct physical meaning only in own subspaces, should be summed carefully. For example, the products $\partial_\mu\varphi^*\cdot\partial^\mu\varphi$ and $\partial_\mu\varphi\cdot\partial^\mu\varphi^*$ are inequivalent in each of subspaces and their contributions to the Hamiltonian differ on the value of the zero-point energy. These contributions having a different sign for contributions with different sign of frequency, sometimes it is offered to cancel with each other. In practice, when both contributions are reduces to the states of particles and antiparticles of the same sign of frequency, their zero-point energies also have the same sign, so as a result the vacuum energy, instead of cancelling, becomes doubled.

As it is known, in the case of harmonic oscillator the coordinates and momenta are *hermitian* operators and the zero-point energy has a clear physical meaning required by the uncertainty relations. In the case of fields hermitian must be the Lagrangian and the canonical variables, i.e. quantum fields arising in the Lagrangian in a bilinear form, in general case are described by *non-hermitian* operators. Moreover, although the Lagrangian should be symmetrical under the basic symmetries of QFT and *the symmetry between the canonical variables is not necessary*. Therefore, at the choosing of the Lagrangians of fields in analogy with a harmonic oscillator, the requirement about the symmetry between the canonical variables is in fact an additional physical hypothesis leading to the divergent zero-point vacuum energy.

The free Lagrangian of a scalar field can be chosen, except for the symmetrized form (2), also in one of following forms:

(a) the standard Lagrangian:

$$L(x)=\left(\partial_\mu\varphi^*\right)\left(\partial^\mu\varphi\right)-m^2\varphi^*\varphi\ , \tag{3}$$

(b) the Lagrangian complex-conjugate to the standard one:

$$L_c(x)=\left(\partial_\mu\varphi\right)\left(\partial^\mu\varphi^*\right)-m^2\varphi\varphi^*\ . \tag{4}$$

As a Lagrangian for a real physical system it can be chosen only an expression which does not lead to a violation of the basic symmetries of QFT and physically unacceptable predictions. Further, as such one we shall consider the standard Lagrangian (3) due to the reasons which will become clear below. A free Hamiltonian and a charge operator corresponding to it are:

$$H = \int \mathrm{d}^3x \left[\left(\partial_t \varphi^*\right)\left(\partial_t \varphi\right) + \left(\nabla \varphi^*\right)\left(\nabla \varphi\right) + m^2 \varphi^* \varphi \right],$$
$$Q = i\int \mathrm{d}^3x \left[\varphi^* \left(\partial_t \varphi\right) - \left(\partial_t \varphi^*\right)\varphi \right]. \tag{5}$$

We see that the Hamiltonian is positive-defined and it seems that there are only positive eigenvalues - a property which usually pointed out in most of textbooks. At the same time, after quantization there appear negative eigenvalues for the negative-frequency solutions due to linear dependence of the energy of quanta on frequency. Because of a clear internal inconsistency in the theory usually this fact has been implicitly ignored. In the previous paper [14] it has been shown that at the changing a sign on energy of a particle there a sign of mass changes also and only then the norm of states remains positive defined. In Application 1 it is shown, how there changes the normalization of the field functions and the sign of a Hamiltonian before the frequency decomposition.

Since after the quantization the energy will change a sign at a changing a sign on frequency, then in the given normalization of field functions there is an exact analogy with the symmetry properties of the harmonic oscillator at a joint changing of signs of energy, mass and frequency [14]. But, having convinced that the paradox is solved for oscillator, we will use the standard normalizations for showing that new results for vacuum energy do not caused by unusual normalization only.

The Lagrangian (3) gives the field equations:

$$\left(\partial_\mu \partial^\mu + m^2\right)\varphi = 0, \qquad \left(\partial_\mu \partial^\mu + m^2\right)\varphi^* = 0. \tag{6}$$

The solutions of these equations are, in particular, plane waves of a frequency k_0 and a wave vector $\mathbf{k}$. Decomposing any field on this full set and using the relativistic relation between the 4-momentum and mass $k^2 = m^2$, it is necessary to take both positive and negative frequency states $k_0 = \pm\omega$, where $\omega \equiv \omega_k = \sqrt{\mathbf{k}^2 + m^2}$. Thus, at a momentum decomposition of field operators usually one separates these two independent sectors:

$$\varphi(x) = \sum_{\mathbf{k}} \left[\tilde{a}\left(\omega, \mathbf{k}\right) e^{-i\omega t + i\mathbf{kx}} + \tilde{a}\left(-\omega, -\mathbf{k}\right) e^{i\omega t - i\mathbf{kx}} \right], \ \sum_{\mathbf{k}} \equiv \int \frac{\mathrm{d}^3 k}{(2\pi)^3 2\omega_k},$$
$$\varphi^*(x) = \sum_{\mathbf{k}} \left[\tilde{a}^*\left(\omega, \mathbf{k}\right) e^{i\omega t - i\mathbf{kx}} + \tilde{a}^*\left(-\omega, -\mathbf{k}\right) e^{-i\omega t + i\mathbf{kx}} \right]. \tag{7}$$

At quantization, the field operators $\varphi(x)$, $\varphi^*(x)$ and the corresponding momenta $\partial_t \varphi^*(x)$, $\partial_t \varphi(\mathrm{x})$ obey the equal time commutation relations:

$$\left[\varphi(\mathbf{x}, t), \partial_t \varphi^*(\mathbf{x}', t)\right] = \left[\varphi^*(\mathbf{x}, t), \partial_t \varphi(\mathbf{x}', t)\right] = i\delta^3\left(\mathbf{x} - \mathbf{x}'\right). \tag{8}$$

As the result, the coefficients of the momentum decomposition become creation-annihilation operators for the particles with the commutation relations, the non-zero of which for the same frequency-sign operators are:

$$\left[\tilde{a}(k),\tilde{a}^{*}(k')\right]=2\omega_{k}(2\pi)^{3}\delta^{3}\left(\mathbf{k}-\mathbf{k}'\right),$$
$$\left[\tilde{a}(-k),\tilde{a}^{*}(-k')\right]=-2\omega_{k}(2\pi)^{3}\delta^{3}\left(\mathbf{k}-\mathbf{k}'\right), \qquad (9)$$

The operators with the opposite sign on frequency should commutate since these two field modes should not interfere. But, this means, that they create or destroy the different kind of particles, i.e. act to the states in the different Fock spaces. Thus, in fact we deal with *an extended space of states* [14] doubled with respect to the positively frequency ones. The such generalized states of the field we can represent in the form:

$$\left|n_{+};n'_{-}\right\rangle=\begin{pmatrix}\left|n_{+}\right\rangle\\ \left|n'_{-}\right\rangle\end{pmatrix}, \qquad (10)$$

where n_{+},n'_{-} are the quantum numbers of the positive- and negative-frequency particles respectively. Then we can introduce the projection operators $P_{\pm}$ with the properties: $P_{\pm}=P_{\pm}^{-1}=P_{\pm}^{2}$, projecting onto the definite frequency-sign states:

$$P_{+}\left|n_{+};n'_{-}\right\rangle=\begin{pmatrix}1&0\\0&0\end{pmatrix}\begin{pmatrix}\left|n_{+}\right\rangle\\ \left|n'_{-}\right\rangle\end{pmatrix}=\begin{pmatrix}\left|n_{+}\right\rangle\\ 0\end{pmatrix}=\left|n_{+}\right\rangle,$$
$$P_{-}\left|n_{+};n'_{-}\right\rangle=\begin{pmatrix}0&0\\0&1\end{pmatrix}\begin{pmatrix}\left|n_{+}\right\rangle\\ \left|n'_{-}\right\rangle\end{pmatrix}=\begin{pmatrix}0\\ \left|n'_{-}\right\rangle\end{pmatrix}=\left|n'_{-}\right\rangle. \qquad (11)$$

The creation-annihilation operators acting in definite frequency-sign subspaces, now can be constructed in the form:

$$a(k)=P_{+}^{-1}\tilde{a}(k)P_{+}, \qquad a^{*}(k)=P_{+}^{-1}\tilde{a}^{*}(k)P_{+},$$
$$a(-k)=P_{-}^{-1}\tilde{a}(-k)P_{-}, \qquad a^{*}(-k)=P_{-}^{-1}\tilde{a}^{*}(-k)P_{-}. \qquad (12)$$

where $a(k)\equiv a(\omega,\mathbf{k}),\ a(-k)\equiv a(-\omega,-\mathbf{k})$. Notice, that now the operators without a tilde have the matrix form:

$$a(k)=\begin{pmatrix}\tilde{a}(k)&0\\0&0\end{pmatrix}, \qquad a^{*}(k)=\begin{pmatrix}\tilde{a}^{*}(k)&0\\0&0\end{pmatrix},$$
$$a(-k)=\begin{pmatrix}0&0\\0&\tilde{a}(-k)\end{pmatrix}, \qquad a^{*}(-k)=\begin{pmatrix}0&0\\0&\tilde{a}^{*}(-k)\end{pmatrix}. \qquad (13)$$

The non-zero commutation relations for these operators are:

$$\left[a(k),a^{*}(k')\right]=2\omega P_{+}(2\pi)^{3}\delta^{3}\left(\mathbf{k}-\mathbf{k}'\right),$$
$$\left[a(-k),a^{*}(-k')\right]=-2\omega P_{-}(2\pi)^{3}\delta^{3}\left(\mathbf{k}-\mathbf{k}'\right). \qquad (14)$$

All other commutators, including the commutators for the different frequency-sign operators, there vanish [14].

Let's notice that because of the negative sign of the second commutator in (14) there was aroused a serious difficulty – the negativity norm of the states that led to the negative probabilities. As it has been shown in previous paper [14], this problem is resolved also and the norms of states will be positive if to take into account correctly the changing a sign of mass at the changing a sign of energy (Appendix 1.)

The vacuum state for the free field $\left|0;0\right\rangle$ is defined as a state without particles of both signs on energy:

$$a(\pm k)\left|0;0\right\rangle = 0, \tag{15}$$

while one particle states are defined as:

$$\left|k;0\right\rangle = a^*(k)\left|0;0\right\rangle, \quad \left|0;-k\right\rangle = a^*(-k)\left|0;0\right\rangle. \tag{16}$$

Thus, after the momentum decomposition, we obtain the expressions:

$$\begin{aligned} H &= \sum_{\mathbf{k}} \omega_k \left[a^*(k)a(k) + a^*(-k)a(-k) \right], \\ Q &= \sum_{\mathbf{k}} \left[a^*(k)a(k) - a^*(-k)a(-k) \right]. \end{aligned} \tag{17}$$

Notice, that the Hamiltonian H_0 and the charge Q are symmetrical under the change a sign on frequency: $\omega \to -\omega$.

The number operators $N_\pm$ for the positive and negative frequency particles:

$$N_+ = \sum_{\mathbf{k}} a^*(k)a(k), \quad N_- = -\sum_{\mathbf{k}} a^*(-k)a(-k), \tag{18}$$

are conserved separately:

$$\left[N_\pm, H\right] = 0. \tag{19}$$

Then the charge operator can be written in the form:

$$Q = N_+ + N_-, \tag{20}$$

which means the total number of particles of both signs on energy.

1.2 *CPT*-transformations of operator products

In the standard QFT the causality conditions require the backward in time propagating for the negative-frequency particles. Therefore, from the *T*-symmetry of the states follows that the relations for the negative-frequency operators should be the same as for the time-reversed positive-energy ones. The T operation for the matrix elements of the products of boson operators is defined as [3-4]:

$$\begin{aligned} &T\left\langle \omega,\mathbf{k}\right| A(\omega,\mathbf{k}) B(\omega',\mathbf{k}') \left|\omega',\mathbf{k}'\right\rangle T^{-1} = \\ &= \left\langle \omega',-\mathbf{k}'\right| B^+(\omega',-\mathbf{k}') A^+(\omega,-\mathbf{k}) \left|\omega,-\mathbf{k}\right\rangle. \end{aligned} \tag{21}$$

It is *nonlinear* operation and includes the hermitian conjugation of the operators and the wave functions, which not only replaces the initial and final states, but also *transposes* the operators in the operator products.

Therefore, if a negative-frequency particle, propagating backward in time, we identify by a positive-frequency antiparticle, propagating forward in time, the standard replacement of the operators (28), yielded directly in the field operators, does not provide a correct time-reversal in the bilinear under the fields expressions for observables. Our problem, therefore, is to provide full and correct time-reversal of the operator products combined with their charge conjugation.

Notice that the time-reversal is generally defined for the matrix elements since it includes the transposition of the initial and final states. For the boson fields and the bilinear combinations of fermionic operators this is insufficient and, for simplicity, the *CP*-transformations can be yielded for the operators only. However, in the case of some fermionic operator products this feature should be taken into account.

Let us consider the standard definitions of the discrete transformations and their combinations applied to the creation-annihilation operators [3-4]:

(1) the parity operator (P) leads only to the changing a sign on momenta:

$$P\left[a(\omega,\mathbf{k})\right]P^{-1} = a(\omega,-\mathbf{k}); \tag{22}$$

(2) the charge conjugation (C) transforms operators of particles to the operators of charge-conjugate them antiparticles b and b^* of the same sign on energy:

$$\begin{aligned} C\left[a(\omega,\mathbf{k})\right]C^{-1} &= b(\omega,\mathbf{k}), \\ C\left[a^*(\omega,\mathbf{k})\right]C^{-1} &= b^*(\omega,\mathbf{k}). \end{aligned} \tag{23}$$

Thus, at the combined operation (*CP*) the operator products transform as:

$$\begin{aligned} CP\left[a^*(\pm\omega,\mathbf{k})a(\pm\omega',\mathbf{k}')\right]P^{-1}C^{-1} &= b^*(\pm\omega,-\mathbf{k})b(\pm\omega',-\mathbf{k}'), \\ CP\left[b^*(\pm\omega,\mathbf{k})b(\pm\omega',\mathbf{k}')\right]P^{-1}C^{-1} &= a^*(\pm\omega,-\mathbf{k})a(\pm\omega',-\mathbf{k}'). \end{aligned} \tag{24}$$

(3) The T operation, defined in Eq.(21), acts on the products of bosonic creation-annihilation operators such that it complex conjugate and transposes the operators in the operator products:

$$\begin{aligned} T\left[a^*(\omega,\mathbf{k})a(\omega',\mathbf{k}')\right]T^{-1} &= a^*(\omega',-\mathbf{k}')a(\omega,-\mathbf{k}), \\ T\left[b^*(\omega,\mathbf{k})b(\omega',\mathbf{k}')\right]T^{-1} &= b^*(\omega',-\mathbf{k}')b(\omega,-\mathbf{k}); \end{aligned} \tag{25}$$

(4) finally, a combined CPT-transformation of the bilinear expressions reads:

$$\begin{aligned} &TCP\left[a^*(\omega,\mathbf{k})a(\omega',\mathbf{k}')\right]P^{-1}C^{-1}T^{-1} = TC\left[a^*(\omega,-\mathbf{k})a(\omega',-\mathbf{k}')\right]C^{-1}T^{-1} = \\ &= T\left[b^*(\omega,-\mathbf{k})b(\omega',-\mathbf{k}')\right]T^{-1} = b^*(\omega',\mathbf{k}')b(\omega,\mathbf{k}). \end{aligned} \tag{26}$$

The same relations take place for the operator products of the negative-frequency particles and their charge-conjugate ones:

$$TCP\left[a^*(-\omega,\mathbf{k})a(-\omega',\mathbf{k}')\right]P^{-1}C^{-1}T^{-1} = b^*(-\omega',\mathbf{k}')b(-\omega,\mathbf{k}), \tag{27}$$

The fact, that the CPT-transformations not only replace the operators to the charge-conjugate ones, but also *transpose the operators in their products*, should be taken into account at quantization of fields and below corresponding improvements of the quantization rules will be performed.

1.3 The charge-conjugation symmetry violation paradox in the standard QFT

In the standard treatment of QFT it has been postulated a direct identification of the negative-frequency operators by positive-frequency operators b and b^* of charge-conjugate particles, i.e. antiparticles, in field operators φ, φ^* [3-4]:

$$\begin{aligned} a^*(-k) &= Ca(k)C^{-1} = b(k), \\ a(-k) &= Ca^*(k)C^{-1} = b^*(k), \end{aligned} \tag{28}$$

and, then the exclusion of the negative-frequency operators has been supposed to be completed.

As the result, the Hamiltonian and the charge operator have been written as:

$$H = \sum_{\mathbf{k}} \omega_k \left[a^*(k)a(k) + b(k)b^*(k) \right],$$
$$Q = \sum_{\mathbf{k}} \left[a^*(k)a(k) - b(k)b^*(k) \right]. \tag{29}$$

Then, they have been rewritten by using commutators as:

$$H = \sum_{\mathbf{k}} \omega_k \left[a^*(k)a(k) + b^*(k)b(k) \right] + H_0$$
$$Q = \sum_{\mathbf{k}} \left[a^*(k)a(k) - b^*(k)b(k) \right] + Q_0. \tag{30}$$

These expressions contain a divergent zero-point energy $H_{(0)}$ and a zero-point charge $Q_{(0)}$ of the vacuum with all their problems. Thus, at the replacements (28) the zero-point energy arises not only in the symmetrized, but in the standard Hamiltonian also. Below we shall show that here the *C*-symmetry of the theory is broken too.

Since at the charge conjugation the operators transform as in (23), after the charge conjugation of H_0 and Q we obtain not their former expressions, but the new operators H_{0C} and Q_C, expressed through charge-conjugate operators:

$$H_C \equiv C H C^{-1} = \sum_{\mathbf{k}} \omega_k \left[b^*(k)b(k) + a(k)a^*(k) \right],$$
$$Q_C \equiv C Q C^{-1} = \sum_{\mathbf{k}} \left[b^*(k)b(k) - a(k)a^*(k) \right]. \tag{31}$$

Further, we take into account that a Hamiltonian of charge conjugation symmetric fields does not vary at that transformations, while a charge changes a sign:

$$H = H_C, \quad Q = -Q_C. \tag{32}$$

Due to independence of the oscillatory modes, these conditions hold for any mode and they give us, using (29), following relationships between the operator products:

$$a^*(k)a(k) + b(k)b^*(k) = b^*(k)b(k) + a(k)a^*(k),$$
$$a^*(k)a(k) - b(k)b^*(k) = -b^*(k)b(k) + a(k)a^*(k). \tag{33}$$

By adding and subtracting these two equalities, we obtain following relationships:

$$a^*(k)a(k) = a(k)a^*(k),$$
$$b(k)b^*(k) = b^*(k)b(k). \tag{34}$$

Thus, by postulating (28) we come to a paradoxical result that for obeying the charge conjugation symmetry conditions (32) the creation-annihilation operators should commutate. This means that for the Lagrangian (3) the standard identification of the different frequency sign operators in (28) contradicts to the *C*-invariance of the theory and, therefore, is inadmissible.

Simplest method for the restoring the charge-conjugation symmetry consists in using of the symmetrized Lagrangian (2). However, in this case there appears the zero-point energy of the vacuum.

In the next section it will be shown, how this paradox can be solved in the case of the standard Lagrangians at more accurate elimination of the negative frequency operators so that the zero-point energy and the zero-point charge do not arise due to the symmetry properties.

1.4 The charge symmetric elimination of the negative-frequency operators

As we see, *CPT*-transformations *do not change a sign on energy* and *relate only the same frequency-sign operator products*. Therefore, it is necessary to define additionally *how we can correctly transform the negative-frequency operator products to the positive-frequency ones*. For this purpose we should find some relations between the operator products having different signs on energy. We could be introduce, in addition to the known *CPT*-transformations, a new symmetry operation (denoting as *E*) changing a sign on energy, which, however, could be require new definitions.

But, fortunately, it is possible to simplify the situation by using the symmetry properties of the Hamiltonian and the charge operator under the time-reversal, or due to the *CPT*-theorem, under the equivalent *CP*-transformations which are simpler. As it will be shown below, it is enough to know *CP*-symmetry properties of the observables (energy and charge) for the expression of all negative-frequency bilinear contributions through the positive-frequency ones.

Since operator products for charged (and *P*-symmetric) fields transform as in (26) and (27), after the *C*-conjugation of H_0 and Q we obtain the operators H_{0C} and Q_C , expressed through operators charge-conjugate to the former ones:

$$H_C \equiv C H C^{-1} = \sum_{\mathbf{k}} \omega_k \left[b^*(k)b(k) + b^*(-k)b(-k) \right],$$
$$Q_C \equiv C Q C^{-1} = \sum_{\mathbf{k}} \left[b^*(k)b(k) - b^*(-k)b(-k) \right]. \tag{35}$$

Then, the *C*-symmetry conditions (32) for H_0 and *Q*, using (17) and (35), lead to following relations between the operator products:

$$a^*(k)a(k) + a^*(-k)a(-k) = b^*(k)b(k) + b^*(-k)b(-k),$$
$$a^*(k)a(k) - a^*(-k)a(-k) = -b^*(k)b(k) + b^*(-k)b(-k). \tag{36}$$

By adding and subtracting these two equalities, we find the required *identities* between the products of opposite sign boson operators:

$$a^*(k)a(k) = b^*(-k)b(-k), \tag{37}$$

$$a^*(-k)a(-k) = b^*(k)b(k). \tag{38}$$

The second identity is only a charge-conjugate form of the first one.

Using the found identities (37) and (38), now we obtain the final expressions for the Hamiltonian and the charge of the scalar field, containing the contributions of the positive-frequency particles and antiparticles only:

$$H = \sum_{\mathbf{k}} \omega_k \left[a^*(k)a(k) + b^*(k)b(k) \right],$$
$$Q = \sum_{\mathbf{k}} \left[a^*(k)a(k) - b^*(k)b(k) \right]. \tag{39}$$

As the result, 4-momentum and 4-current can be expressed in terms of the positive-frequency operators in the form:

$$P_\mu = \sum_{\mathbf{k}} k_\mu \left[a^*(k)a(k) + b^*(k)b(k) \right],$$
$$J_\mu = \sum_{\mathbf{k}} \frac{k_\mu}{\omega_k} \left[a^*(k)a(k) - b^*(k)b(k) \right]. \tag{40}$$

Thus, in the expressions for H_0 and Q, in contrast to the former replacements (28), *the operators of antiparticles appear as the normal-ordered ones without additional hypotheses and the zero-point vacuum energy and zero-point vacuum charge do not arise*.

The case of symmetrized Lagrangians, which are exact analogs of the non-relativistic harmonic oscillator, is considered in Appendix 2, where it is shown that they lead to the divergent zero-point energy and, consequently, for the relativistic fields such Lagrangians are excluded by the existing experiments.

The quantization of a complex vector field is a natural generalization of above proposed procedure for complex scalars field with well-known peculiarities.

1.5 Quantization at spontaneous symmetry breaking

The Hamiltonian of a system of fields at spontaneous symmetry breaking finally represents a Hamiltonian of interacting (massive and/or massless) fields. Here a state with minimal energy is a ground state which represents vacuum after quantization of small fluctuations about it. Therefore, after corresponding “shifting” of fields with nonzero vacuum expectations and equating to zero of energy of the state of minimal energy, *CP*-symmetric quantization of the free Hamiltonian of the system can be performed by the above considered method.

The Lagrangian of the system of $SU(2)$ gauge field A_μ^a and an isodublet of complex scalar field $\tilde{\varphi}, \tilde{\varphi}^*$ has the form:

$$L(x) = \frac{1}{4g} F_{\mu\nu}^a F_a^{\mu\nu} + (D_\mu \tilde{\varphi}^*)(D^\mu \tilde{\varphi}) - \lambda^2 \left(\tilde{\varphi}^* \tilde{\varphi} - \eta^* \eta \right)^2. \tag{41}$$

where $\eta = const$ a complex constant, $D_\mu = \partial_\mu + ig\tau^a A_\mu^a / 2$ and

$$\tilde{\varphi}(x) = \begin{pmatrix} \tilde{\varphi}_1 \\ \tilde{\varphi}_2 \end{pmatrix}, \quad \tilde{\varphi}^*(x) = \left(\tilde{\varphi}_1^*, \tilde{\varphi}_1^* \right). \tag{42}$$

Then the first complex field $\tilde{\varphi}_1$ we exclude by a gauge transformation and for the second field introduce a complex field φ, φ^* with a zero asymptotes at infinity:

$$\tilde{\varphi}_2(x) = \eta + \varphi(x), \quad \tilde{\varphi}_2^*(x) = \eta^* + \varphi^*(x). \tag{43}$$

Then the Lagrangian of the fields transfers to:

$$L(x) = -\frac{1}{2}\left(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a \right)^2 + (\partial_\mu \varphi^*)(\partial^\mu \varphi) - m^2 \varphi^* \varphi - \frac{1}{2} \lambda^2 \left| \eta \right|^2 A_\nu^a A^{a\nu} + L_{\text{int}}. \tag{44}$$

with mass $m = \lambda |\eta|$ for the field φ.

The complex scalar field is quantized further on the above considered method without a vacuum zero-point energy and zero-point charge. Quantization of gauge fields will be considered in the subsequent publications.

Thus, the vacuum of a scalar field does not lead to the diverging zero-point energy and zero-point charge if the spontaneous symmetry breaking happens so that the remained scalar field $\tilde{\varphi}_2$ would remain as complex one and its charge-conjugation symmetry is maintained. In that case the Standard Model predicts not one a scalar boson, but a pair of boson-antiboson with hypercharge, as well as before spontaneous symmetry breaking.

1.6 Quantization of the spinor fields

The momentum decomposition of the spinor fields has the form:

$$\begin{aligned}\psi(\mathrm{x}) &= 2m\sum_{\mathbf{k},\alpha}\left[b_\alpha(k)u_\alpha e^{-i\omega t+i\mathbf{kx}} + b_\alpha(-k)v_\alpha e^{i\omega t-i\mathbf{kx}}\right],\\ \psi^+(\mathrm{x}) &= 2m\sum_{\mathbf{k},\alpha}\left[b_\alpha^+(k)u_\alpha^+ e^{i\omega t-i\mathbf{kx}} + b_\alpha^+(-k)v_\alpha^+ e^{-i\omega t+i\mathbf{kx}}\right].\end{aligned} \tag{45}$$

At quantization the coefficients become creation-annihilation operators with anticommutators, the non-zero of which are:

$$\left\{b_\alpha(\pm k), b_{\alpha'}^+(\pm k')\right\} = \frac{\omega_k}{m}(2\pi)^3\delta^3(\mathbf{k}-\mathbf{k}')\delta_{\alpha\alpha'}P_\pm. \tag{46}$$

As well as in the case of the boson fields, we have the extended space of states, including negative-frequency states as a subspace. As the result, different frequency-sign operators, acting in the different Fock spaces, anticommutate.

The expressions for the Hamiltonian and the charge are:

$$\begin{aligned}H &= i\int d^3x\left[\psi^+\partial_t\psi - (\partial_t\psi^+)\psi\right],\\ Q &= \int d^3x\psi^+\psi,\end{aligned} \tag{47}$$

which, after the substitution of the momentum decompositions, take the form:

$$\begin{aligned}H &= 2m\sum_{\mathbf{k},\alpha}\omega_k\left[b_\alpha^+(k)b_\alpha(k) - b_\alpha^+(-k)b_\alpha(-k)\right],\\ Q &= 2m\sum_{\mathbf{k},\alpha}\left[b_\alpha^+(k)b_\alpha(k) + b_\alpha^+(-k)b_\alpha(-k)\right].\end{aligned} \tag{48}$$

Corresponding charge-conjugate operators are:

$$\begin{aligned}d_\alpha(k) &= Cb_\alpha(k)C^{-1}, \quad d_\alpha(-k) = C\,b_\alpha(-k)C^{-1},\\ d_\alpha^+(k) &= Cb_\alpha^+(k)C^{-1}, \quad d_\alpha^+(-k) = C\,b_\alpha^+(-k)C^{-1}.\end{aligned} \tag{49}$$

Then the charge-conjugate Hamiltonian and charge operator are expressed as:

$$\begin{aligned}H_C &= 2m\sum_{\mathbf{k},\alpha}\omega_k\left[d_\alpha^+(k)d_\alpha(k) - d_\alpha^+(-k)d_\alpha(-k)\right],\\ Q_C &= 2m\sum_{\mathbf{k},\alpha}\left[d_\alpha^+(k)d_\alpha(k) + d_\alpha^+(-k)d_\alpha(-k)\right].\end{aligned} \tag{50}$$

By using the charge conjugation symmetry of the Hamiltonian and the antisymmetry of the charge (32), for any independent field mode we obtain following relations between the operator products:

$$\begin{aligned}b_\alpha^+(k)b_\alpha(k) - b_\alpha^+(-k)b_\alpha(-k) &= d_\alpha^+(k)d_\alpha(k) - d_\alpha^+(-k)d_\alpha(-k),\\ b_\alpha^+(k)b_\alpha(k) + b_\alpha^+(-k)b_\alpha(-k) &= -\,d_\alpha^+(k)d_\alpha(k) - d_\alpha^+(-k)d_\alpha(-k).\end{aligned} \tag{51}$$

By adding and subtracting these two equalities, we obtain the required operator identities for the fermionic operator products:

$$\begin{aligned} b^{+}_{\alpha}(k)b_{\alpha}(k) &= -d^{+}_{\alpha}(-k)d_{\alpha}(-k), \\ b^{+}_{\alpha}(-k)b_{\alpha}(-k) &= -\,d^{+}_{\alpha}(k)d_{\alpha}(k). \end{aligned} \tag{52}$$

These two identities turn to each other at the charge conjugation. As the result, we obtain the final expressions for the Hamiltonian and the charge operator of fermions expressed in terms of the positive-frequency operators only:

$$\begin{aligned} H &= 2m\sum_{\mathbf{k}}\omega_k\left[b^{+}_{\alpha}(k)b_{\alpha}(k) + d^{+}_{\alpha}(k)d_{\alpha}(k)\right], \\ Q &= 2m\sum_{\mathbf{k}}\left[b^{+}_{\alpha}(k)b_{\alpha}(k) - d^{+}_{\alpha}(k)d_{\alpha}(k)\right]. \end{aligned} \tag{53}$$

Thus, at the charge conjugation symmetric quantization of the fermion field with the standard Lagrangian the zero-point vacuum energy and the zero-point vacuum charge also do not arise and the operators in the expressions for observables again are normal-ordered automatically. In general case of chiral fields it is necessary to use, certainly, the combined *CP*-symmetry conditions.

2. *CP*-symmetric quantization in general and the vacuum structure

2.1 The rules of *CP*-symmetric quantization of fields

Thus, due to incomplete time reversal, the standard rules for the operator quantization lead to the zero-point energy and the zero-point charge of fields and, therefore, they should be modified. In the previous sections it has been shown, how we can more accurately eliminate the negative-frequency operator products for the cases of complex boson and spinor fields by using the *CP*-symmetry conditions. Now we can formulate the modified procedures in a general form as the *additional quantization rules*, paying attention only on their difference from the standard ones. Thus, main steps at the *CP-symmetric quantization* of complex relativistic fields (or *CP-quantization*), are:

(1) The Lagrangian of a complex field, due to the existence of the negative-frequency modes, should be taken in the standard form, which is hermitian, but unsymmetrical under the complex conjugate field operators.

(2) The positive- and negative-frequency states form the separate Fock spaces and their direct sum forms an extended space of states with generalized operators having a matrix form and acting in definite frequency sign subspaces.

(3) The momentum decomposition of field operators leads to the creation and annihilation operators with positive- and negative-frequency modes, acting in own subspaces and obeying corresponding (anti)commutation relations;

(4) The creation-annihilation operators for the *CP*-conjugate particles of both signs on energy should be introduced and the observables and their *CP*-conjugate forms should be expressed through the products of creation-annihilation operators of both signs on frequency.

(5) By using the *CP*-(anti)symmetry conditions for the observables (energy and charge), the operator identities relating the products of the negative-frequency operators by the products of the positive-frequency operators should be obtained. By means of these identities all observables can be expressed across the products of the positive-frequency operators for particles and antiparticles.

As it has been shown, only the set of these rules, naturally following from the *T*- and *CP*-symmetries of the theory, allows one to formulate the procedures of quantization in accordance with the *CPT*-symmetry requirements and in the framework of the first principles of QFT.

At last, all these procedures are equivalent to the simple procedure *of normal ordering* of operators in observables, accepted in the standard QFT. Therefore, further we can consider *the procedure of normal ordering as a legal one following from the basic symmetries of the theory*.

2.2 The theorem about vanishing of the zero-point vacuum energy

One of the important results of the previous sections we can formulate in the form of the following theorem:

Theorem. *At the quantization of the CP-symmetric complex relativistic fields with the standard* (*unsymmetrized under the field operators*) *Lagrangians, the creation-annihilation operators in the field's observables are normal-ordered due to the symmetry conditions, and the zero-point vacuum energy and the zero-point vacuum charge do not arise.*

Since the statements of the theorem have been checked up for each of the basic complex relativistic fields (scalar, vector and spinor), we can consider the theorem as proved by an induction, while its proving in a general form will be considered in the forthcoming publications.

Intuitively the zero-point fluctuations of fields have been associated by the fact that the fields are presented as sets of oscillators at the quantization of which there appear the zero-point energies. In the paper [14] it has been shown, that in the non-relativistic oscillatory potential there are the positive zero-point energies for both for particles and antiparticles. Thus, the reason for the occurrence of zero-point energies is not the transition from the negative-frequency operators to the positive-frequency ones, but only the presence in a Hamiltonian of the symmetrized products of the identity frequency sign operators. Analogs of the Hamiltonian of the non-relativistic oscillator are the Hamiltonians symmetrized under the field operators, considered in the Appendix 2, here the zero-point energies arises. Therefore, in the case of the unsymmetrized Lagrangians the zero-point energies of fields do not appear, and this is the case of the standard Lagrangians of QFT.

The fact, that at the *CP*-symmetric quantization of relativistic fields the problems with the zero-point energy of the vacuum disappear, means that in the case of the free fields QFT is a consistent theory leading to the physically correct answers for the correctly formulated questions without additional hypotheses.

2.3 The experiments excluding the zero-point fluctuations of fields

If, as it is widely accepted [3,4], to accept the real existence of zero-point energy of vacuum, then there appears obvious contradictions of this hypothesis with the experiments. It is well known, that such effects as the Lamb shift, the anomalous magnetic moment and the Casimir effect can be fully explained by the interaction of an electron by the zero-point fluctuations of vacuum fields and, at the same time, they can be described exactly as the results of the interaction of particles by real sources. Here it is important to clearly understand the fact, that the presented in the literature two explanations of these effects are not *alternative* ways of description of the same phenomena, but they are *two independent mechanisms* contributing to the observable effects additively and the values of which should be added.

If we consider the existing experiments in more details and in each case try to separate a contribution of the zero-point vacuum fluctuations which cannot be explained as the contribution of real sources, in each case we found out that such contribution does not exist. Here we consider some of them:

1. If there exist the zero-point vacuum fluctuations of the electromagnetic field, *S* and *P* levels of the Hydrogen-like atoms should be shifted differently [2-4] and the relative shift must be very close to the observed value of the Lamb shift. In addition, the zero-point vacuum fluctuations should give the same contributions to the anomalous magnetic moments of particles as contributions of the loop and the external sources [5]. However, the experiments and exact calculations in all cases show that there it is enough the interactions of charges only with the virtual particles (loop contributions) and/or fields of real sources, without reference to the interactions with the zero-point vacuum fields. Since the reality of the contributions of the virtual particles and the external fields of the sources is obvious, it is clear, that only they are observing really in the experiments and thus, these experiments exclude the existence of the zero-point fluctuations of the electromagnetic field with very high accuracy.

2. The widely accepted treatment of the Casimir effect is that in this case we directly measure the forces generating by the zero-point fluctuations of vacuum fields. However, it is also well known, that the Casimir effect can be exactly described across the contributions of fields of real sources – atoms in solid states [6-10]. The well-known Lifshitz's theory and its modifications [7-9] are based on the consideration of properties of fluctuating electromagnetic fields of atoms at a finite temperature, including the fields generating by *vibrations of atoms* (not vacuum fields!) in crystals at zero temperature and this theory completely explain the Casimir force as a form of the van der Waals like forces. Therefore, the measurements of the Casimir effect also exclude the existence of the zero-point fluctuations of the electromagnetic field’s vacuum [10].

3. The infinite zero-point energy of vacuum leads to the infinite cosmological constant. In fact, according to observations, cosmology gives so low value for the vacuum energy density that from the point of view of particle physics it may be taken as a vanishing one.

Thus, all these facts show, that only the contributions of the fields of real sources completely explains each of known effects and that, therefore, the experiments with high accuracy exclude the contribution of the zero-point fluctuations of vacuum fields. The experimental evidences about the lack of the zero-point fluctuations at the same time are the experimental verifications of the formulated theorem about vanishing of the zero-point energy of the relativistic fields. More detailed discussion of the existing experiments will be presented in forthcoming publications.

Conclusion

Thus, only the spectrum of the relativistic fields with the Lagrangians symmetrized under the field operators is similar to the spectrum of the non-relativistic harmonic oscillator where there is the zero-point energy. It is shown that the vacuum of relativistic fields with the standard Lagrangians, unsymmetrical under the complex-conjugate fields, does not contain the zero-point energy and zero-point charge due to the *T*- and *CP*-symmetry conditions. The standard rules of the operator quantization of free fields are modified for the coincidence with basic discrete symmetries of QFT.

The fact that the zero-point vacuum fluctuations do not exist for the *CP*-symmetric fields is confirmed by the existing experiments with very high accuracy. The experiments, therefore, exclude the Lagrangians symmetrized for the complex-conjugate relativistic fields and allow one to use only the standard Lagrangians without zero-point vacuum fluctuations.

When Dirac's sea for fermions at the negative-energy states has been replaced by Feynman's universal time reversal treatment of antiparticles, it was disappeared the numerous intrinsic contradictions of the former treatment. It is shown, that due to the same time-reversal symmetry we do not need in the hypothesis about the zero-point vacuum fluctuations with all its problems and contradictions. At the same time, this fact legalizes the procedure of the normal ordering as a simplified method for the taking into account the requirements of the basic symmetries of the theory.

At spontaneous symmetry breaking the vacuum zero-point energy and zero-point charge of do not appear only if the remained scalar field is complex one and in this case the Standard Model predicts a pair boson-antiboson with hypercharge.

The presented in the paper natural exclusion from the formalism of QFT of the divergent zero-point vacuum energy and zero-point vacuum charge without additional hypothesis or mathematical tricks is the first step for a final consistent formulation of QFT. In the forthcoming papers it will be shown, that at a correct definition of integrations over time and energy, QFT also does not contain the divergent vacuum loop contributions due to the freezing of all fluctuations at the Planck energy, where the gravitational time dilation and redshifts effectively cut the loop integrals, including those which contribute to the vacuum energy.

The important consequence of these result will be the fact, that all known vacuum effects, including smallness of the cosmological constant, can be naturally explained in the framework of standard principles of QFT without new hypothesis and there no contradiction between particle physics and cosmology.

Appendixes

1. Normalization of fields with different sign Hamiltonians

At the usual normalization of fields there is a strange fact that the positive-defined Hamiltonian after the quantization leads to the negative energy for the negative-frequency solutions because of a linear dependence on frequency. Below it will be shown that there exists another normalization of the fields when the Hamiltonian does not positive-defined initially and at the changing of a sign on mass the energy also changes a sign, as it is required by causality.

Lagrangian for the charged scalar field φ can be written in the form:

$$L(x) = \mu\left[(\partial_\mu \varphi^*)(\partial^\mu \varphi) - m^2 \varphi^* \varphi\right]. \qquad (54)$$

where a new normalization of the field functions has been used differing from the standard one by a factor $\mu^{1/2}$ in analogy with the harmonic oscillator [12]. Here the new parameter μ is mass of "atoms of a chain" and m plays a role of the frequency of oscillations about a fixed center. For the corresponding system of real scalar fields φ_1 and φ_2 we have:

$$L(x)=\frac{\mu}{2}\left[(\partial_\mu\varphi_1)(\partial^\mu\varphi_1)+(\partial_\mu\varphi_2)(\partial^\mu\varphi_2)-m^2(\varphi_1^2+\varphi_2^2)\right] \tag{55}$$

The Hamiltonian and the charge operator at this normalization have the form:

$$\begin{aligned} H &= \int \mathrm{d}^3x\left[\frac{1}{\mu}\pi\pi^* + \mu(\nabla\varphi^*)(\nabla\varphi)+\mu m^2\varphi^*\varphi\right], \\ Q &= i\int \mathrm{d}^3x(\varphi^*\pi^*-\pi\varphi), \quad \pi=\mu\partial_t\varphi^*. \end{aligned} \tag{56}$$

Field equations do not vary, while the decompositions of the field operators read:

$$\begin{aligned} \varphi(x) &= \sum_{\mathbf{k}}\left[\tilde{a}(k)e^{-i\omega t+i\mathbf{kx}}+\tilde{a}(-k)e^{i\omega t-i\mathbf{kx}}\right], \ \sum_{\mathbf{k}} \equiv \int\frac{\mathrm{d}^3k}{(2\pi)^3 2\mu\omega_k}, \\ \varphi^*(x) &= \sum_{\mathbf{k}}\left[\tilde{a}^*(k)e^{i\omega t-i\mathbf{kx}}+\tilde{a}^*(-k)e^{-i\omega t+i\mathbf{kx}}\right]. \end{aligned} \tag{57}$$

The equal time commutators

$$[\varphi(\mathbf{x},t),\pi(\mathbf{x}',t)]=\left[\varphi^*(\mathbf{x},t),\pi^*(\mathbf{x}',t)\right]=i\delta^3(\mathbf{x}-\mathbf{x}'), \tag{58}$$

give the positive defined commutators for the creation-annihilation operators:

$$\left[\tilde{a}(\pm k),\tilde{a}^*(\pm k')\right]=2\mu\omega_k(2\pi)^3\delta^3(\mathbf{k}-\mathbf{k}'). \tag{59}$$

Thus, the parameter μ as the «mass of atoms of a chain» appears in the Hamiltonian (56) linearly and at a joint changing of signs of frequency, energy and mass μ the norm of states of both signs of frequency are positively defined.

Transition to a usual normalization of the field functions and their generalized momenta is made not at taking $\mu\to 1$, but at more correct replacement $\mu\to\pm 1$ depending on the signs of the frequency and the energy.

2. The fields with the symmetrized Lagrangians

For complex fields it is enough if the Lagrangian is hermitian as in (3). However, we can suppose also that the Lagrangian be symmetrized under field operators as in (2). The Hamiltonian and the charge operator then have the form:

$$\begin{aligned} H &= \frac{1}{2}\int \mathrm{d}^3x\left[\{\partial_t\varphi^*,\partial_t\varphi\}+\{\nabla\varphi^*,\nabla\varphi\}+m^2\{\varphi^*,\varphi\}\right], \\ Q &= \frac{i}{2}\int \mathrm{d}^3x\left[\{\varphi^*,\partial_t\varphi\}-\{\partial_t\varphi^*,\varphi\}\right], \end{aligned} \tag{60}$$

where $\{A,B\}=AB+BA$. The momentum decomposition gives for Hamiltonians:

$$H = \frac{1}{2}\sum_{\mathbf{k}} \omega_k \left[a^*(k)a(k) + a(k)a(k) + a^*(-k)a(-k) + a(-k)a^*(-k) \right],$$
$$H_C = \frac{1}{2}\sum_{\mathbf{k}} \omega_k \left[b^*(k)b(k) + b(k)b^*(k) + b^*(-k)b(-k) + b(-k)b^*(-k) \right]. \tag{61}$$

The charge operator and its charge conjugate form are:

$$Q = \frac{1}{2}\sum_{\mathbf{k}} \left[a^*(k)a(k) + a(k)a^*(k) - a^*(-k)a(-k) - a(-k)a^*(-k) \right],$$
$$Q_C = \frac{1}{2}\sum_{\mathbf{k}} \left[b^*(k)b(k) + b(k)b^*(k) - b^*(-k)b(-k) - b(-k)b^*(-k) \right], \tag{62}$$

Then the charge conjugation symmetry conditions lead to the operator identities:

$$a^*(k)a(k) + a(k)a^*(k) = b^*(-k)b(-k) + b(-k)b^*(-k)$$
$$a^*(-k)a(-k) + a(-k)a^*(-k) = b^*(k)b(k) + b(k)b^*(k). \tag{63}$$

Using these identities, we obtain the expressions:

$$H = \frac{1}{2}\sum_{\mathbf{k}} \omega_k \left[a^*(k)a(k) + a(k)a^*(k) + b^*(k)b(k) + b(k)b^*(k) \right] =$$
$$= \sum_{\mathbf{k}} \omega_k \left[a^*(k)a(k) + b^*(k)b(k) \right] + H_{(0)}, \tag{64}$$

$$Q = \frac{1}{2}\sum_{\mathbf{k}} \left[a^*(k)a(k) + a(k)a^*(k) - b^*(k)b(k) - b(k)b^*(k) \right] =$$
$$= \sum_{\mathbf{k}} \left[a^*(k)a(k) - b^*(k)b(k) \right]. \tag{65}$$

Here $H_{(0)}$ is the zero-point energy and $Q_{(0)}$ is the zero-point charge appearing in the theories with the symmetrized Lagrangians, as for the harmonic oscillator [14].

Thus, the symmetrized Lagrangians of fields lead to the divergent zero-point vacuum energy and contradict to the existing experiments. Therefore, only the standard (unsymmetrized) Lagrangians of QFT, considered above in the paper, are admissible as free ones on the zero-point energy.

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