Theoretical Physics, Astrophysics and Cosmology ISSN 1819-9135; EISSN 1991-3915
Vol. **9**, No 1, p. 1 – 4, v1, 6 May 2014

Online: TPAC: 4874-034 v1, 6 May 2014 DOI: 10.9751/TPAC.4874-034

A model of the closed universe gravitating in 4-space

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Abstract

Models of the closed Universe as a thin 3-sphere in 4-space, gravitating along 3-sphere's radius, are reformulated in a new form, in which at a local environment of an observer the non-relativistic dynamics of a ball is reproduced with a correct energy conservation condition. Corresponding evolution equations for dust matter and radiation in the 3-sphere are obtained and their observational consequences are studied. It is shown that the closed models in 4-space also lead to the "Miniverse" model with a highly oscillating curve for the "distance modulus – redshift" relation.

PACS: *04.20.Cv, 98.80.-k, 98.80.Jk 95.30.Sf, 97.60.Lf, 98.35.Jk, 98.54.-h, 98.80.-k, 04.60.-m*
Key words: cosmological models, redshift, extra dimension

Content

Introduction

In the previous papers new and more consecutive formulations of models of relativistic cosmology on the basis of accounting of some well known observational facts and inevitable theoretical restrictions have been presented. They are the existence of large gravitationally-bound regions, such as galaxy clusters, where it occurs a stasis of wavelength of passing photons [1], requirements of relativistic kinematics, especially in the closed model [2] and promoting the energy conservation in a local environment of any observer [3].

This analysis has shown that former confrontations with observations of the models without dark energy were premature since did not include the number of physical requirements and effects, every one of which essentially influences to the observational predictions of models. Particularly, the progress in the closed models, which from the theoretical point of view always have been considered as most attractive ones, is that part of former arguments against them practically have disappeared.

However, the accounting of energy conservation conditions in 3-space has led to already new and unusual problem - to a conclusion that the radius of the closed Universe should be very small ("mini-Universe" or "Miniverse") [3]. In that case real objects are only what are in a small vicinity of the observer, and more remote ones appear as their repeated images from earlier epochs. Large-scale homogeneity and isotropy then find a natural explanation, but here the periodicity effects should appear. Are these consequences

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shortcomings of the model or contrary advantages and predictions, are the questions requiring further investigations.

In the present paper this analysis will be continued and it will be studied the consequences of the existence of the closed Universe as 3-sphere only at embedding into 4-space and, therefore, gravitating in 4-space. It will be shown, that in this most consecutive formulation of the closed model the property of smallness of radius of the Universe and the prediction for "Miniverse" keep the force.

The main relations of the model are presented in Sections 1-2 and in Section 3 the observational consequences are considered.

1. The evolution equation from Einstein's equations in 4-space

Considering the closed model of relativistic cosmology, further we will suppose that the Universe is 3-sphere of radius a embedded into the real 4-dimensional space and that its evolution is described in a rest-frame of the centre of 3-sphere in terms of world time t.

The solutions of the Einstein equations $G_{ik} = \kappa T_{ik}$ for the metrics of a thin dust 3-sphere in 4-space are well-known and they lead to the evolution equation:

$$\frac{\dot{a}^2}{c^2} = \frac{a_g^2}{a^2} - 1, \tag{1}$$

where a_g is the gravitational radius of 3-sphere. As at modification of the Friedman equation for correct matching with non-relativistic dynamics of an expanding ball at vicinity of an observer [4], we subtract from this equation its value at stopping at the maximal radius $a = a_m$ and obtain a physically correct form of the evolution equation:

$$\frac{\dot{a}^2}{c^2} = \frac{a_g^2}{a^2} - \frac{a_g^2}{a_m^2}. \tag{2}$$

The speed of expansion and parametre H_0 in the model are equal to:

$$\dot{a} \simeq c\frac{a_g}{a}\sqrt{1 - \frac{a^2}{a_m^2}}, \tag{3}$$

$$H_0 = \frac{\dot{a}_0}{a_0} \simeq c\frac{a_g}{a_0^2}\sqrt{1 - \frac{a_0^2}{a_m^2}} = \frac{c}{a_0}\frac{\eta}{b}\sqrt{1-b^2}, \tag{4}$$

where $b = a_0 / a_m$ $\eta = a_g / a_m$ and

$$a_0 = \frac{c}{H_0}\eta b^{-1}\sqrt{1-b^2}. \tag{5}$$

At taking into account the effects of relativistic kinematics [2], it is necessary to express the evolution equation (1), where the speed is a derivative on proper time, in terms of world time t defined as

$$d\tau = dt\sqrt{1 - \frac{a_g^2}{a^2} - \frac{\dot{a}^2}{c^2}}, \tag{6}$$

after which it takes the form:

$$\frac{\dot{a}^2/c^2}{1-a_g^2/a^2-\dot{a}^2/c^2}=\frac{a_g^2}{a^2}-1. \tag{7}$$

Then, for obeying to the local energy conservation condition, we subtract from this equation its value at stopping at maximal radius $a=a_m$ and we obtain a corrected evolution equation:

$$\frac{\dot{a}^2/c^2}{1-a_g^2/a^2-\dot{a}^2/c^2}=\frac{a_g^2}{a^2}-\frac{a_g^2}{a_m^2}, \tag{8}$$

which gives an expression for radial speed in terms of world time:

$$\frac{\dot{a}^2}{c^2}=\frac{a_g^2}{a^2}\cdot\frac{(1-a^2/a_m^2)(1-a_g^2/a^2)}{1+(1-a^2/a_m^2)a_g^2/a^2}. \tag{9}$$

For radiation instead of (8) we have the evolution equation:

$$\frac{\dot{a}^2/c^2}{1-a_g^2/a^2-\dot{a}^2/c^2}=\frac{a_{ge}^3}{a^3}-\frac{a_{ge}^3}{a_m^3}, \tag{10}$$

where $a_{rg}^3\sim\rho_r(t_0)a_0^4=\rho_r(t_m)a_m^4$ with corresponding change of other relations.

2. Observable consequences of the model

From the equation for trajectory of a radially propagating light:

$$ds^2=c^2dt^2-a^2(t)\cdot d\chi^2=0 \tag{11}$$

then follows:

$$\chi_z=c\int_{a_z}^{a_0}\frac{da}{a}\cdot\frac{dt}{da}=\frac{1}{a_g}\int_{a_z}^{a_0}\frac{da}{\sqrt{1-a^2/a_m^2}}=\frac{1}{\eta}\left(\arcsin(b)-\arcsin\left(b\frac{a_z}{a_0}\right)\right), \tag{12}$$

or

$$\sin\chi_z=\sin\left[\frac{1}{\eta}\arcsin\left(b\sqrt{1-b^2\frac{a_z^2}{a_0^2}}-b\frac{a_z}{a_0}\sqrt{1-b^2}\right)\right]. \tag{13}$$

The redshift z of wavelength λ_r at reception with respect to wavelength λ_e at radiation and the relation with the scale factors (in the nonrelativistic case) are defined as:

$$\frac{\lambda_r}{\lambda_e}=1+z,\quad \frac{\lambda_r}{\lambda_e}=\frac{a_0}{a_z}, \tag{14}$$

which after substitution into (13) lead to

$$\sin\chi_z=\sin\left\{\frac{1}{\eta}\arcsin\left[\frac{b\sqrt{1-b^2}}{1+z}\left(\sqrt{1+\frac{2z+z^2}{1-b^2}}-1\right)\right]\right\}. \tag{15}$$

Since $1/\eta=a_m/a_g\gg1$, it is a very quickly oscillating function.

In the case $a_0\ll a_m$, i.e. $b\ll1$, we would obtain:

$$\sin\chi_z=\sin\left(\frac{1}{\eta}\arcsin\frac{bz}{1+z}\right)\simeq\sin\left(\frac{a_0}{a_g}\cdot\frac{z}{1+z}\right), \tag{16}$$

In the limit $z \to 0$ we have $a_0 \sin\chi_z \to c/H_0$, which allows further to normalize the distance modulus.

Taking into account that the photometric distance is equal to $d_p = a_0 \sin\chi$, for apparent luminosity l_F and photometric distance $d_{p,0}$ we obtain the expressions:

$$l_F = \frac{L}{4\pi a_0^2 \sin^2\chi_z} \cdot \frac{1}{(1+z)^2}, \tag{17}$$

$$d_p = a_0 |\sin\chi| \cdot (1+z) = 10^{-5+(m-M)/5} Mnc, \tag{18}$$

which for the distance modulus give the formula:

$$\mu = 5\lg\left[a_0(1+z)\cdot|\sin\chi|\right] + 25 = 5\lg\left[\eta b^{-1}\sqrt{1-b^2}\cdot(1+z)|\sin\chi|\right] + A, \tag{19}$$

where $A = 5\lg(c/H_0) + 25$. Substitution of Eq. (16) into (19) gives a new "distance modulus – redshift" relation for the "Miniverse" model:

$$\mu = 5\lg\left[\eta b^{-1}\sqrt{1-b^2}\cdot(1+z)\left|\sin\left\{\frac{1}{\eta}\arcsin\left[\frac{b\sqrt{1-b^2}}{1+z}\left(\sqrt{1+\frac{2z+z^2}{1-b^2}}-1\right)\right]\right\}\right|\right] + A. \tag{20}$$

In the model this formula is valid up to enough high values of z while the assumptions about dust matter and insufficiency of the gravitational redshift remain be valid.

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