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Models of the Universe with strongly conserved energy of matter

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Abstract

It is shown that earlier in the Friedmann models requirements of nonrelativistic dynamics of a dust ball, concerning the energy conservation, have not been obeyed, which then led to inconsistency at formulation of relativistic models too. It is proposed a method for reformulation of models of relativistic cosmology when at small distances they naturally coincide by the nonrelativistic model of the dust ball. As the result, in such modified model some of former problems of the Friedmann models do not arise.

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Content

Introduction

The modern standard paradigm in cosmology, presented in reviews and textbooks, is based on the Friedmann models completed, for describing observations, with necessary amount of a dark matter and a dark energy [1,2]. It is supposed that disagreement of standard models with the observations before introduction of unobserved components of energy are not related with shortcomings of the theoretical basis of the accepted paradigm, but caused only with insufficiency of the observational data about the matter and energy density in the real Universe.

Contrary to this direction based on confidence on the foundations of the accepted paradigm, other approaches, also based on general relativity (GR), have been developed which cast doubt on some assumptions of the standard paradigm. At more careful analysis there really appear gaps in justification and in the first attempts of their completion a share of dark components in the energy balance or essentially decreases, or even disappear. In particular, in the previous paper [3] a formulation of relativistic cosmology on the basis of dynamics of a homogeneous ball has been studied and a revision of the Friedmann model following from the requirements of relativistic kinematics has been considered.

Another aspect of models of the Universe is connected with the fact that in local physics at the solution of the field equations it has been supposed disappearance of surface integrals "at infinity", where curvature created by compact sources should disappear. This conventional picture of local physics has been applied globally to cosmological models too,

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while the cosmological principle assumes homogeneity at any distances. In fact in cosmology the constants of integration of relativistic models will be defined more correctly not at “infinity” distances, but locally near the observation point by “matching” with non-relativistic models on the basis of Newtonian gravity.

It, in turn, imposes restrictions on cosmological models, in particular, that at small distances it should be reproduced a nonrelativistic energy conservation condition for a participating in expansion particle.

In the present paper it will be shown that earlier in the Friedmann models requirements of nonrelativistic dynamics of a dust ball, concerning the energy conservation, have not been obeyed, which then led to inconsistency at formulation of the relativistic model too.

In the paper it will be proposed a method of reformulation of models of relativistic cosmology after which at small distances they will naturally coincide by the nonrelativistic model of the dust ball. As the result, there will not arise some of former problems of the Friedmann models

The main relations of the model are presented in Sections 1-2, in Section 3 the relativistic kinematics is taken into account and in Section 4 some observational consequences are discussed.

1. Non-relativistic dynamics of a homogeneous dust ball in 3-sphere

The general-relativistic evolution equation for the homogeneous and isotropic Universe, as it is known, simply may be derived from the Newtonian model of evolution of a homogeneous dust ball of radius *r*, volume $V(r)$ and mass $m=\rho\cdot V(r)$, where ρ is mean matter density [1,2]. For this purpose the ball should be appropriately big to be homogeneous and to expand, but also not too big so that speed of surface with respect to the centre of the ball remained nonrelativistic.

Let's consider at first an *isolated* ball expanding in empty space and an object of unit mass comoving to the surface of the ball. In the Newtonian theory total energy *E* of this object conserves and is equal, up to a constant *C* defining zero of energy, to the sum of kinetic and potential energies. If the sum of kinetic and potential energies of a particle on the surface is negative, the ball will stop after some time at a maximal radius r_m .

The evolution equation for the particle on the surface we obtain in *two stages*:

а) at first we find expression for *the total energy* of the particle;

б) then, by using *the energy conservation condition*, exclude the constant *C*, by choosing one of states where we know the speed exactly.

Simplest choice is a moment of stopping of the ball at $r=r_m$, when we have:

$$E=\frac{1}{2}\dot{r}^2-\frac{Gm}{r}+C=-\frac{Gm}{r_m}+C, \tag{1}$$

where $\dot{r}=dr/dt$ is velocity with respect to ball’s centre, G is the gravitational constant. These relations give us *the evolution equation*:

$$\frac{1}{2}\dot{r}^2-\frac{Gm}{r}=-\frac{Gm}{r_m}, \tag{2}$$

which may be written in a more convenient form as:

$$\frac{\dot{r}^2}{c^2}=\frac{r_g}{r}-\frac{r_g}{r_m}, \tag{3}$$

where $r_g = 2Gm/c^2$. From three unknown constants of the system in Eq. (1) - E, C and r_m - here the first two are excluded and only r_m is remained. In terms of densities we obtain:

$$\dot{r}^2 = \frac{4\pi}{3}\cdot 2G\cdot(\rho r^2 - \rho_m r_m^2). \tag{4}$$

The ball with maximal radius of expansion formally is analogue of the closed model of relativistic cosmology, whereas the flat model corresponds to $r_m \to \infty$. Thus, it is enough to consider the closed model only, by considering the flat model formally as its limiting case. In the closed Universe $r(t) = a(t)\sin\chi$ and the considered ball is a segment of a 3-sphere, angular volume of which is equal to $V_\chi = \pi(2\chi - \sin 2\chi)$. Turning to variables of 3-sphere, we can consider evolution of a hemisphere $\chi = \pi/2$ with larger at every moment a value of r equal to $r = a$, at which the *evolution equation* looks like:

$$\frac{\dot{a}^2}{c^2} = \frac{a_{g,1/2}}{a} - \frac{a_{g,1/2}}{a_m}, \tag{5}$$

where $a_{g,1/2} = 2GM_{1/2}/c^2$, and $M_{1/2} = \pi^2 G\rho_0 a_0^3/c^2$ is mass of matter in the hemisphere. In terms of densities this evolution equation takes the form:

$$\dot{a}^2 = \pi^2\cdot 2G\cdot(\rho a^2 - \rho_m a_m^2). \tag{6}$$

In expanding space filled by homogeneous dust matter a ball *is not isolated* and the matter out of the ball creates in it independent on coordinates, but time-dependent gravitational potential, value of which changes during expansion. In particular, to a particle on the border of two hemispheres with $\Delta\chi = \pi/2$ a gravitational attraction of both hemispheres acts equally and the gravitational force, decelerating the expansion, doubles. As a result, the right part of Eq.(5) also doubles and we obtain the final evolution equation for 3-sphere:

$$\frac{\dot{a}^2}{c^2} = \frac{a_g}{a} - \frac{a_g}{a_m}. \tag{7}$$

In terms of densities it has the form:

$$\dot{a}^2 = 2\pi^2\cdot 2G\cdot(\rho a^2 - \rho_m a_m^2), \tag{8}$$

where $a_g = 2GM/c^2$, and $M = 2M_{1/2} = 2\pi^2 G\rho_0 a_0^3/c^2$ is mass of matter in 3-sphere.

2. The general-relativistic evolution equation for 3-sphere

The standard Friedmann equation

$$\frac{\dot{a}^2}{c^2} = \frac{a_g}{a} - 1, \tag{9}$$

following from the Einstein equations, differs from (7) by the replacement

$$\frac{a_g}{a_m} \to 1. \tag{10}$$

Earlier Eq. (9) has been formally obtained from dynamics of the nonrelativistic ball by artificial replacement of the formula (2), following from the energy conservation condition, by physically meaningless equation [1,2]:

$$\frac{1}{2}\dot{r}^2 - \frac{Gm}{r} = -\frac{c^2}{2}. \tag{11}$$

In fact, Eqs. (9) and (11) also are inconsistent and inadmissible not only in the framework of the Newtonian theory, which it would be possible to carry on its approximate nature, but in the framework of GR too. In GR a radius of an expanding ball should be *larger* than its gravitational radius $r_g < r \le r_m$, that gives restrictions $a_g < a \le a_m$, whereas from Eqs. (9) and (11) follow from (7) and (2) at assumptions $a_m = a_g$ and $r_m = r_g$, which are physically inadmissible.

This means that or the standard Friedmann metrics, or the energy-momentum tensor used in Einstein's equations for obtaining (9), contain certain unrealized in the nature (non-physical) assumptions. Corrections of the metrics will be considered in another paper and here we will consider a correction of expressions for the energy-momentum related with protection of the energy conservation.

As we see, Eq. (11) contains a certain constant which presented in Eq. (1) in the general form and it showed a point where we count zero of total energy E. It is expressed through E and a value of the potential at $r = r_m$, which then allows one to turn from Eq. (1) to Eq. (2) satisfying the energy conservation condition. At the same time, Eq.(11) violates the energy conservation condition due to some constant. Thus, we have a task to restore the energy conservation in this case too, after which a modified form of Eq.(11), probably, gain a physical sense.

For this purpose it is natural to apply the same method as at transition from Eq. (1) to Eq. (2), i.e. we not fix "by hand" a constant of energy for fitting to Eq. (9), but use the fact that at the stopping point $r = r_m$, where $\dot{r}^2(r_m) = 0$, the right hand side of Eq. (11) is expressed through the value of the potential in this point. Thus, we come to Eq. (2) and by that we completely restore the energy conservation in the system. Instead of Friedman's equation (9) we unambiguously come to the evolution equations for 3-sphere (7) or (8).

A similar procedure of correct definition of the constant of energy in the ball's evolution equation has been used earlier (see [2], the equation (1.2.4)), but only a value of the total energy has been taken not at the stopping time, but at "present" $t = t_0$ when one can measure $\dot{r}_0$. This had given:

$$\frac{\dot{r}^2}{c^2} = \frac{r_g}{r} - \left(\frac{r_g}{r_0} - \frac{\dot{r}_0^2}{c^2}\right) = \frac{4\pi}{3} \cdot 2G \cdot \left[\rho r^2 - \left(\rho_0 - \rho_c\right) r_0^2\right], \tag{12}$$

where r_0 is the present radius, the matter density $\rho(t)$ and critical density ρ_c have been defined as:

$$\rho(t) = \frac{3}{4\pi} \cdot \frac{m}{r^3(t)}, \quad \rho_c = \frac{3}{4\pi} \cdot \frac{H_0^2}{2G}. \tag{13}$$

It is important, that in the closed model the last bracket in (12) should be positive, i.e $\rho_0 > \rho_c$. In fact an observable value of ρ_0 had appeared more than on an order less than ρ_c and thus this fact has been considered as main argument against the closed model.

Despite improvement of one discrepancy, nevertheless, this analysis contained additional discrepancies and was not consecutive, so further refinements and corrections are required which then will change conclusions too, including the last one.

At first, in the closed model at the stopping time $t = t_m$ with $\dot{r}_m^2 = 0$ Eq. (12) gives the relationship between the constants r_0 and r_m :

$$\frac{r_g}{r_0} - \frac{\dot{r}_0^2}{c^2} = \frac{r_g}{r_m}, \tag{14}$$

which should be obeyed at any moment. After inverse substitution of this relation into Eq. (12), the latter takes the form of Eq. (3) with all consequences considered in Section 1. Thus, the ball's evolution equation (12) leads not to the standard Friedmann model with Eq. (9), but to the model on the basis of Eq. (7), different from the standard one.

Secondly, if from Eq. (12) one passes to 3-sphere as a whole, instead of Eq. (13) it is necessary to enter already other definition of density:

$$\tilde{\rho}(t) = \frac{1}{2\pi^2} \cdot \frac{M}{a^3(t)}, \quad \tilde{\rho}_c = \frac{1}{2\pi^2} \cdot \frac{H_0^2}{2G} = \frac{2}{3\pi} \rho_c. \tag{15}$$

Then an improved form of the evolution equation (12) for 3-sphere looks like as:

$$\frac{\dot{a}^2}{c^2} = 2\pi^2 \cdot 2G \cdot \left[\tilde{\rho} a^2 - \left(\tilde{\rho}_0 - \tilde{\rho}_c \right) a_0^2 \right]. \tag{16}$$

Since the improved critical density almost five times less than former one $\tilde{\rho}_c \simeq \rho_c / 4.71$, whereas in full density of matter $\tilde{\rho}_0 = \rho_{0,\mathrm{b}} + \rho_{0,\mathrm{d}}$ the observable density of energy of baryons $\rho_{0,\mathrm{b}}$ remains as former, for justification of the closed model the dark matter density $\rho_{0,\mathrm{d}}$ should only few times exceed $\rho_{0,\mathrm{b}}$ that is quite acceptable and can coincide with observational restrictions.

3. Relativistic dynamics of a ball

We start by the Schwarzschild's exact solution for a gravitational field on the surface of a ball of mass *m*. Here world time $\tilde{t}$ is time of a rest frame of ball's centre. This time will coincide with proper time on the surface expanded up to very large radius $r_m \to \infty$.

For energy of an object of unit mass comoving to the surface, then we have expression (in this section $c = 1$):

$$\frac{\sqrt{1 - r_g / r}}{\sqrt{1 - v^2}} = \sqrt{1 - r_g / r_m}, \quad v(r) = \frac{\dot{r}}{\sqrt{1 - r_g / r}}. \tag{17}$$

By introducing notation $\eta(r_g, r_m) = 1/(1 - r_g / r_m)$, we rewrite this relation in the form:

$$\dot{r}^2 = \frac{r_g}{r}\left(1 - \frac{r}{r_m}\right)\left(1 - \frac{r_g}{r}\right)\eta(r_g, r_m). \tag{18}$$

One can define the maximal radius from data for certain time:

$$r_m = r_g \cdot \left(\frac{r_g}{r_0} - v_0^2 \right)^{-1} (1 - v_0^2). \tag{19}$$

By substituting $r(\tilde{t}) = a(\tilde{t}) \sin\chi$ and considering a hemisphere $\chi = \pi / 2$, when $r(\tilde{t}) = a(\tilde{t})$, $m = M / 2$ and $V_{\chi=\pi/2} = \pi^2$, we come to the relativistic evolution equation:

$$\frac{da^2}{d\tilde{t}^2} = \frac{a_g}{a}\left(1-\frac{a}{a_m}\right)\left(1-\frac{a_g}{a}\right)\eta(a_g, a_m). \tag{20}$$

At early periods the Universe has a large expansion speed and for objects with high z the accounting of relativistic effects is required. Thus, the proper time of ball's centre $\tilde{t}$ we express through global world time of 3-sphere's centre t (further $\dot{a} \equiv da/dt$):

$$d\tilde{t} = dt\sqrt{1-\dot{a}^2}. \tag{21}$$

Then the linear element on the 3-sphere takes the form [3]:

$$ds^2 = dt^2 - da^2 - a^2(t)\cdot d\Omega_{(3)}^2 = dt^2(1-\dot{a}^2) - a^2(t)\cdot d\Omega_{(3)}^2. \tag{22}$$

By substituting (21) into (5), we come to the evolution equation for the Universe taking into account the relativistic effects of 3-sphere's radial speed of expansion:

$$\frac{\dot{a}^2}{1-\dot{a}^2} = \frac{a_g}{a} - \frac{a_g}{a_m}. \tag{23}$$

The expression for the relativistic factor then looks like

$$\sqrt{1-\dot{a}^2} = \frac{1}{\sqrt{1-\eta^2 + a_g/a}}. \tag{24}$$

where $\eta \equiv \sqrt{a_g/a_m}$. From (24) it follows that at epochs with $a \gg a_g$ the relativistic effects become insufficient.

4. Observable consequences of the new evolution equation

Thus, we have the new evolution equation (7) for 3-sphere following from Einstein's equations at their adjustment at small distances with the nonrelativistic dynamics of the ball with correctly defined total energy. It differs from Friedman's equation (9) by the last term and leads to a new class of cosmological models with observable consequences.

a) *Flat model.*

At first we will consider a standard limiting case $a_m \gg a$ when in (7) it is possible to neglect the last term, believing $\eta \to 0$:

$$\frac{\dot{a}^2}{c^2} \simeq \frac{a_g}{a}. \tag{25}$$

From the equation for trajectory of a radially propagating light:

$$ds^2 = c^2dt^2 - a^2(t)\cdot d\chi^2 = 0 \tag{26}$$

then follows:

$$\chi_z = c\int_{a_z}^{a_0}\frac{da}{a}\cdot\frac{dt}{da} = \frac{1}{\sqrt{a_g}}\int_{a_z}^{a_0}\frac{da}{\sqrt{a}} = \frac{2}{\sqrt{\eta_0}}\left(1-\sqrt{a_z/a_0}\right). \tag{27}$$

where $\eta_0 = a_g/a_0$. The redshift z of wavelength at reception λ_r in comparison with wavelength at radiation λ_e and relation with the scale factors (in the nonrelativistic case) are defined as:

$$\frac{\lambda_r}{\lambda_e} = 1 + z, \quad \frac{\lambda_r}{\lambda_e} = \frac{a_0}{a_z}, \tag{28}$$

which after substitution gives

$$\chi_z = \frac{2}{\sqrt{\eta_0}}\left(1 - \frac{1}{\sqrt{1+z}}\right). \tag{29}$$

The apparent and absolute luminosities l, L of sources at lack of expansion would be connected with photometric distance as $d_p \simeq a\chi$ as $l = L / 4\pi d_p^2$. Through apparent and absolute magnitudes m, M they are expressed as $l = 10^{-m/2.5} \cdot 2.52 \cdot 10^{-5}$ erg/sm^{2}sek and $L = 10^{-M/2.5} \cdot 3.02 \cdot 10^{35}$ erg/sec. The expansion leads to redshifting of energy and frequency of arrived photons by factor $1 + z$. The standard expression for apparent luminosity then has the form:

$$l_F = \frac{L}{4\pi d_{p,0}^2} \cdot \frac{1}{(1+z)^2}. \tag{30}$$

The photometric distance $d_{p,0}$ is thus equal to:

$$d_{p,0} = a_0 \cdot (1+z) = 10^{-5+(m-M)/5} Mpc, \tag{31}$$

from which for the distance modulus $\mu \equiv m - M = 5\lg(d_{p,0}) + 25$ we obtain the expression:

$$\mu = 5\lg\left[a_0(1+z)\cdot\chi_z\right] + 25 = 5\lg\left[2\left(1 + z - \sqrt{1+z}\right)\right] + A, \tag{32}$$

where $A \equiv 5 \cdot \lg(c / H_0) + 25$, $H_0 = \dot{a}_0 / a_0$ and $c / H_0 = a_0 / \sqrt{\eta_0}$.

b) *A model of the closed "Miniverse".*

Let's consider now a “mini-Universe”, or “Miniverse” case when the radius is small and the most of observable sources are repeated images. In this case speed of expansion is small $\dot{a}_0 \sim \eta c \ll c$, and the effects of relativistic kinematics considered in [3], can be neglected for all observable objects. Let's pass to studying of observable consequences of this new version of relativistic cosmology.

By introducing the notation $b = a_0 / a_m$, from (7) we have:

$$a_0 = \frac{c}{H_0}\eta\sqrt{b^{-1} - 1}, \quad a_m = \frac{a_0}{b}. \tag{33}$$

The first conclusion is that since $a_0 \sim \eta \cdot c / H_0$ and $\eta < 10^{-2}$, the modern radius of the Universe a_0 appears at least on two order less than the “Hubble radius” c / H_0. The Universe in that case appears as very little and we deal with the *mini-Universe* model. Real objects then are in radius about 100 Mpc, and all other observable objects should be then multiple images which have appeared at round-the-world ray of light before detection. Homogeneity and isotropy the Universe out of the main sphere then can be naturally explained.

From (7) and (26) we obtain:

$$\chi_z = c\int_{a_z}^{a_0} \frac{da}{a} \cdot \frac{dt}{da} = \frac{1}{\eta}\int_{a_z}^{a_0} \frac{da}{\sqrt{a(a_m - a)}} = \frac{1}{\eta}\left[\arcsin\left(1 - \frac{2a_z}{a_m}\right) - \arcsin\left(1 - \frac{2a_0}{a_m}\right)\right]. \tag{34}$$

$$\sin\chi_z = \sin\left\{\frac{1}{\eta}\arcsin\left[2\sqrt{b(1-b)}\,\frac{a_z}{a_0}\left[\frac{a_0}{a_z}-1-(1-2b)\left(\sqrt{\frac{a_0/a_z-b}{1-b}}-1\right)\right]\right]\right\}. \quad (35)$$

It gives

$$\sin\chi_z = \sin\left\{\frac{1}{\eta}\arcsin\left[\frac{2\sqrt{b(1-b)}}{1+z}\left[z-(1-2b)\left(\sqrt{1+\frac{z}{1-b}}-1\right)\right]\right]\right\}. \quad (36)$$

where $1/\eta$ a large number, but nevertheless at $z \to 0$ we have:

$$a_0 \sin\chi_z \to c/H_0. \quad (37)$$

The photometric distance is $d_p = a\sin\chi$ and for the apparent luminosity we have the expression:

$$l_F = \frac{L}{4\pi a_0^2 \sin^2\chi_z}\cdot\frac{1}{(1+z)^2}. \quad (38)$$

This gives for the photometric distance $d_{p,0}$:

$$d_{p,0} = a_0\left|\sin\chi\right|\cdot(1+z) = 10^{-5+(m-M)/5} Mnc, \quad (39)$$

which for the distance modulus leads to the expression:

$$\begin{aligned}\mu &= 5\lg\left[a_0(1+z)\cdot\left|\sin\chi\right|\right]+25 = \\ &= 5\lg\left[\eta\sqrt{b^{-1}-1}\cdot(1+z)\left|\sin\chi\right|\right]+A,\end{aligned} \quad (40)$$

Substitution Eq. (36) into (40) gives new “distance modulus–redshift” relation for the “Miniverse” model.

At first sight confrontation with observations should yield negative result since such strong effects of periodicity in the distance modulus were not revealed. However, if the model corresponds to reality, these effects may be somehow smoothed by other factors and some smooth periodicity along the mean line may takes place. In this case it will be interesting studying of thin structure of the dependence $\mu(z)$ which can contain such screen periodic component in addition to the smooth one.

More detailed comparison with observation and discussion of the considered models will be carried out in the subsequent publications.

References

1. Weinberg S. (2008) *Cosmology*. Oxford.U.P.
2. Zeldovich Ya. B., Novikov I.D. (1983) *Relativistic Astrophysics, 2. The Structure and Evolution of the Universe*. Ch.U.P.
3. Zakir Z. (2013) *Theor. Phys. Astroph.& Cosmol*. 8(1), 16; doi: 10.9751/TPAC.4518-031