

Are Strings Thermostrings?¹

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Abstract

In the method of thermostring quantization the time evolution of point particles at finite temperature kT is described in a geometric manner. The temperature paths of particles are represented as closed (thermo)strings, which are swept surfaces in a space-time-temperature manifold. The method makes it possible a new physical interpretation of superstrings IIA and heterotic strings as point particles in a thermal bath with Planck temperature.

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Introduction

In the history of physics the replacement of local fundamental objects by nonlocal ones has occurred numerous times. Newton's local particles with nonlocal interactions were replaced by Maxwell's fields. In relativity theory and in quantum physics point particles were reintroduced as fundamental objects along with fields.

Last time the strings have been introduced as hypothetical nonlocal objects with local interactions [1]. One direction for the development of this hypothesis would be to extend it into branes and further abandon the locality of primary objects of the theory. Another direction would be to restore the locality of primary objects by reducing strings back to point particles and local fields with specific behavior.

The method of thermostring quantization [2-3] is a simple step in the latter direction. Here strings are considered as thermostrings, i.e., point particles in $(D-2)$ -dimensional space with a thermal length. Due to the coupling of the coordinates of these particles with a temperature degree of freedom, their time evolution in a D-dimensional space-time-temperature manifold is described by the string formalism. This method does not introduce any new object into the physics. Instead it considers new properties of old objects. The physical reason for nonlocality is Gibbs ensemble averaging, and the length of the thermostring is nothing but the thermal "wordline" of the point particle [4].

A similar story happened in quantum theory with Schrödinger's wave mechanics. At first the wave function was considered as a description of real waves in physical space-time the same as electromagnetic waves. The understanding that these are really

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probability waves of point particles in configurational space was achieved through M. Born's statistical interpretation of the wave function.

Analogously, the thermostring quantization is a statistical interpretation of strings. It explains the appearance of the string behavior of particles at Planck distances naturally by replacing the space-time fluctuations at the Planck temperature with a thermal bath. In this approach space-time has a finite temperature, which can be taken into account in a geometrical manner by introducing a temperature degree of freedom as an additional dimension of the physical manifold.

1. What is the string?

In conventional string theory [1], strings are considered to be fundamental one-dimensional objects in physical space. The direct introduction of such nonlocal structures into physics leads to some conceptual difficulties concerning the measurement of points of strings, intrinsic dynamics, relativistic causality, etc. Strings are alien to the geometry of space-time and the reduction of strings into space-time properties is a nontrivial problem.

The generalizations of the string theory do not change the physical nature of strings. All them lead to the string theory at some stage and we still have the old strings as nonlocal objects in space.

2. What is the thermostring?

The density matrix $\rho(\mathbf{r}, \Delta\beta)$ for a nonrelativistic particle at a finite temperature $kT = 1/\Delta\beta$, after factorization $\Delta\beta = \beta - \beta_0$, can be represented as a transition amplitude $\rho(\mathbf{r}, \beta; \mathbf{r}_0, \beta_0)$ for pure states $\psi_i(\mathbf{r}, \beta)$ in the space-temperature manifold (\mathbf{r}, β) [Fe]:

$$\begin{aligned} \rho(\mathbf{r}, \mathbf{r}_0; \Delta\beta) &= \sum_i \exp(-E_i \Delta\beta) \psi_i(\mathbf{r}) \psi_i^*(\mathbf{r}_0) = \\ &= \sum_i \psi_i(\mathbf{r}, \beta) \psi_i^+(\mathbf{r}_0, -\beta_0) = \rho(\mathbf{r}, \beta; \mathbf{r}_0, -\beta_0) \end{aligned} \quad (1)$$

where $\Delta\beta = \beta - \beta_0$, E_i - is the energy of the particle, \mathbf{r} are d -dimensional space coordinates. Here wave functions $\psi_i(\mathbf{r}, \beta)$ are the pure states of the particle with the Hamiltonian H in (\mathbf{r}, β) -manifold:

$$\psi_i(\mathbf{r}, \beta) = \exp(-H\beta) \psi_i(\mathbf{r}), \quad (2)$$

$$\psi_i^*(\mathbf{r}, \beta) = \psi_i^*(\mathbf{r}) \exp(H\beta),$$

$$\int \psi_i^*(\mathbf{r}, \beta) \psi_j(\mathbf{r}, \beta) d\mathbf{r} = \delta_{ij}. \quad (3)$$

and:

$$\psi_i(\mathbf{r}, \beta) = \int d\mathbf{r}_0 \rho(\mathbf{r}, \beta; \mathbf{r}_0, \beta_0) \psi_i(\mathbf{r}_0, \beta_0), \quad (4)$$

The partition function $Z(\Delta\beta)$ is defined as:

$$Z(\Delta\beta) = \int d\mathbf{r} \rho(\mathbf{r}, \beta; \mathbf{r}_0, \beta_0) |_{\mathbf{r}(\beta)=\mathbf{r}(\beta_0)}. \quad (5)$$

The (inverse) temperature parameter of evolution β here can be considered as a geometrical dimension of the physical manifold in addition to space and time. Here β is

limitless $-\infty \leq \beta \leq \infty$, but the intervals $\Delta\beta$ are restricted by the condition $0 \leq \Delta\beta \leq 1/kT$.

The density matrix as the β -evolution transition amplitude can be represented through an infinity set of intermediate states [4]:

$$\begin{aligned} \rho(\mathbf{r}, \beta; \mathbf{r}_0, \beta_0) &= \int d\mathbf{r}(\beta_1) \dots d\mathbf{r}(\beta_n) \psi(\mathbf{r}, \beta) \psi^+(\mathbf{r}_n, -\beta_n) \psi(\mathbf{r}_n, \beta_n) \times \dots \\ &\dots \times \psi^+(\mathbf{r}_1, -\beta_1) \psi(\mathbf{r}_1, \beta_1) \psi^+(\mathbf{r}_0, -\beta_0) \end{aligned} \quad (6)$$

We can transfer all $\psi_i(\mathbf{r}_k, \beta_k)$ to the left hand side and all $\psi_i^+(\mathbf{r}_k, -\beta_k)$ to the right hand side. Then we can compose wave functionals Ψ_i, Ψ_i^+ as the products of an infinity number ($n \rightarrow \infty$) intermediate state wave functions of particles:

$$\begin{aligned} \rho(\mathbf{r}, \beta; \mathbf{r}_0, \beta_0) &= \int d\mathbf{r}(\beta_1) \dots d\mathbf{r}(\beta_n) \{ \psi(\mathbf{r}, \beta) \psi(\mathbf{r}_n, \beta_n) \dots \psi(\mathbf{r}_1, \beta_1) \} \times \\ &\times \{ \psi^+(\mathbf{r}_n, -\beta_n) \dots \psi^+(\mathbf{r}_1, -\beta_1) \psi^+(\mathbf{r}_0, -\beta_0) \} \\ &= \sum_{\mathbf{r}(\beta_0)}^{\mathbf{r}(\beta)} \int D\mathbf{r}(\beta') \Psi[\mathbf{r}(\beta'); \beta, \beta_0] \Psi^+[\mathbf{r}(\beta'); -\beta, -\beta_0]. \end{aligned} \quad (7)$$

Here $D\mathbf{r}(\beta)$ is the path integration measure. The wave functional Ψ describes the states of the temperature path:

$$\Psi[\mathbf{r}(\beta'); \beta_n, \beta_1] = \psi(\mathbf{r}_n, \beta_n) \psi(\mathbf{r}_{n-1}, \beta_{n-1}) \dots \psi(\mathbf{r}_1, \beta_1), \quad (8)$$

$$\Psi^+[\mathbf{r}(\beta'); -\beta_n, -\beta_1] = \psi^+(\mathbf{r}_1, -\beta_1) \dots \psi^+(\mathbf{r}_{n-1}, -\beta_{n-1}) \psi^+(\mathbf{r}_n, -\beta_n),$$

$$\int D\mathbf{r}(\beta') \Psi_i^+[\mathbf{r}(\beta'); -\beta, -\beta_0] \Psi_j[\mathbf{r}(\beta'); \beta, \beta_0] = \delta_{ij} \quad (9)$$

We see that the temperature path of the point particle (β world line) can be represented as some one dimensional physical object in space-time-temperature manifold with the wave functional Ψ . We will call this object as a *thermostring* [2].

In the general case, the wave functional Ψ must be symmetrized under permutations of points of the temperature paths or thermostrings. These permutations depend on the type of statistics of the particles in the Gibbs ensemble (bosonic or fermionic) and they determine the type of statistics of thermostrings. In ordinary string theory such permutations are impossible since ordinary strings are treated as continuous one dimensional objects in physical space.

The time evolution of the density matrix can be described by summing all of the surfaces which are swept by any initial temperature path in time up to any final temperature path. This circumstance allows us to introduce the thermostring representation of the quantum statistical mechanics of particles or *the thermostring quantization* of particles at finite temperatures.

Taking into account time evolution formulas for wave functions $\psi_i(\mathbf{r}, \beta)$:

$$\psi_i(\mathbf{r}, \beta, t) = \exp(-iHt) \psi_i(\mathbf{r}, \beta), \quad (10)$$

we can obtain the time evolution expression for the wave functional Ψ .

In the (\mathbf{r}, β) manifold we have $(d+1)$ -dimensional coordinates \mathbf{q} with components (\mathbf{r}, β) . Time derivatives of these vectors are space-temperature velocities of particles. They are separated into longitudinal components and transverse to the temperature path components:

$$\begin{aligned}\partial \mathbf{q} / \partial t &= \mathbf{v} = \mathbf{v}_\perp + \mathbf{v}_\parallel, \quad \mathbf{v}_\perp = \mathbf{v} - \mathbf{k}(\mathbf{q}' \mathbf{v}), \\ \mathbf{q}' &= \partial \mathbf{q} / \partial \beta, \quad \mathbf{k} = \mathbf{q}' / \mathbf{q}'^2.\end{aligned}\quad (11)$$

The longitudinal components of \mathbf{v}_\parallel also have two parts. The first part leads to the collective motion of the thermostring as a whole object with synchronous displacements of all points of the thermostring. In the case of a single thermostring this displacements can be disregarded as the zero mode. The second part of the longitudinal velocity leads to permutations of the points of the thermostring, eg., the replacement of neighboring points. This permutation does not contribute to the energy of the thermostring because of the indistinguishability of the particles in Gibbs ensemble. In ordinary string theory the exclusion of \mathbf{v}_\parallel from the Lagrangian is one of the difficulties of the theory [5], whereas in the case of thermostrings this is a natural and necessary process.

So, we have following time evolution expression for the wave functional:

$$\Psi_i[\mathbf{q}(\bar{\beta}, t), \beta, \beta_0; t - t_0] = \exp \left[-\frac{i t}{\Delta \beta} \int_{\beta_0}^{\beta} d\beta' H(\mathbf{v}_\perp^2, \beta') \right] \Psi_i[\mathbf{q}(\beta'); \beta, \beta_0; t_0]. \quad (12)$$

The action function for thermostrings is:

$$S[x] = -\frac{m}{\Delta \beta} \int d\beta dt \sqrt{1 - \mathbf{v}_\perp^2} \quad (13)$$

where $(d+2)$ -vector x^μ have the components $x^\mu(\mathbf{q}, t) = x^\mu(\mathbf{r}, \beta, t)$. It can be shown [BN] that after introduction of world sheet parameters τ, σ and substitutions:

$$\begin{aligned}t &= t(\tau, \sigma), \quad d\beta = d\sigma \sqrt{x'^2}, \\ \dot{x}^\mu &= \partial x^\mu / \partial \tau, \quad x' = \partial x / \partial \sigma, \\ dtd\sigma &= \frac{\partial(t, \sigma)}{\partial(\tau, \sigma)} d\tau d\sigma = \dot{t} d\tau d\sigma,\end{aligned}\quad (14)$$

$$\partial \mathbf{q} / \partial t = \dot{\mathbf{q}} / \dot{t}, \quad \partial \mathbf{q} / \partial \sigma = \mathbf{q}' - \dot{\mathbf{q}} \cdot (t' / \dot{t}),$$

this expression leads to the Nambu-Goto action for the relativistic thermostring:

$$S[x] = -\gamma \int d\sigma d\tau \sqrt{(\dot{x}x')^2 - \dot{x}^2 x'^2} \quad (15)$$

Here $\gamma = m / \Delta \beta$, σ and τ are the world sheet coordinates of thermostring.

We see that in the thermostring representation of the quantum statistical mechanics of particles there exist reparametrizational symmetry $\sigma' = f(\sigma, \tau)$, $\tau' = \phi(\sigma, \tau)$, just as in string theories. This action is fully relativistically invariant if $\Delta \beta$ and the limits of β -integration are invariants. At ordinary temperatures this is impossible, but if we take the relativistically invariant Planck temperature T_p as the limiting temperature of thermostrings with $\Delta \beta = 1 / kT_p$, we have an invariant action function.

The action function and the periodicity conditions $x^\mu(\sigma, \tau) = x^\mu(\sigma + \pi, \tau)$ lead to the equations for $x_\mu(\sigma, \tau)$ with solutions:

$$\begin{aligned}
x^\mu(\sigma, \tau) &= x_R^\mu(\sigma, \tau) + x_L^\mu(\sigma, \tau), \\
x_{R,L}^\mu(\sigma, \tau) &= \frac{1}{2}x_0^\mu + \frac{1}{2}l^2 p^\mu(\tau \mp \sigma) + \frac{i\alpha'}{2} \sum_n \frac{1}{n} \alpha_{n,\pm}^\mu e^{-2in(\tau \mp \sigma)}
\end{aligned} \tag{16}$$

where $n \neq 0$, x_0^μ , p^μ - constants, $l^2 = 1/\pi\alpha'$. Then the operators for dynamical variables can be constructed and they lead to the Virasoro algebra and to the spectrum of states identical with the closed bosonic string case. In the case of the fermionic particles the fermionic thermostrings with the periodicity conditions for fermionic degrees of freedom can be obtained.

We can describe the time evolution by surface integration along the world surface which the thermostrings are swept in time. The result is identical with the Polyakov surface integral for closed strings.

3. Are strings thermostrings?

We can replace the space-time fluctuations at Planck distances with a thermal bath with Planck temperature. When we take the influence of this thermal bath on the particle's behavior into account, we come to the thermostring quantization scheme. At Planck distances particles must be described as thermostrings and we can interpret superstrings as thermostrings, i.e., as point particles in a Planckian thermal bath. This interpretation preserves all of the achievements of ordinary string theory and at the same time excludes some of the conceptual difficulties associated with the introduction of nonlocal objects with unmeasurable intrinsic structure.

Thermostring interpretation of strings leads to the following general consequences:

a) One of the dimensions of the string theory manifold is the (inverse) temperature dimension and therefore the dimension of space is $d = 8$ which, together with the time and temperature degrees of freedom, combine into the critical dimension $D = d + 2 = 10$. This fact is important in the compactification of six dimensions which can be represented as $5 + 1$ (5 space and 1 temperature).

b) In the amplitudes of strings for the initial and final states only closed strings can appear as observable physical states.

c) The charge of particles must be distributed along the length of the thermostring.

Among superstring theories only theories of closed strings satisfy these conditions and we may conclude that only superstrings IIA (in the case of neutral particles) and heterotic strings (in the case of charged particles) [GSW] can be interpreted as thermostrings. This means that the thermostring interpretation selects only one theory for each type of particle from the family of string theories.

We can describe thermostring interactions simultaneously in particle, statistical ensemble, and string languages. The factorization of one statistical ensemble into two ensembles or the merging of two ensembles into one are physically clearer and simple procedures than the cutting or gluing of rigid strings in conventional string theory.

We can perform the thermostring quantization of physical strings as one-dimensional objects in physical space and as a result obtain a theory of *thermomembranes*. If we perform the thermostring quantization of physical p-branes we shall obtain a theory of (p+1)-*thermobranes*, i.e., the dimensionality of initial objects of the theory increases by one. In the treatment of p-branes the thermostring representation can be combined with M-Theory methods if we interpret one of its eleven dimensions as temperature.

Thus, the thermostring quantization can be the simplest and natural way to understand the modification of local theories at Planck distances and temperatures.

References

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