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Quantization of relativistic fields without zero-point energy
2. Vector and spinor fields

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Abstract

The new method of *C*- and *CP*-symmetric quantization of complex fields is applied for spinor, complex vector and electromagnetic fields. It is shown that the constraints imposed by requirements of discrete symmetries on bilinear products of creation-annihilation operators lead to operators of observables in a normal-ordered form without a zero-point energy and a zero-point charge.

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Content

Introduction

In the previous papers [1] a new version of the method of quantization of systems with charge-conjugation (*C*-) symmetry has been proposed. Unlike the initial version of the method [2], the creation-annihilation operators of positive-frequency states have been used only. Quantization of a system, represented as a set of oscillators with complex generalized coordinates, has been considered and then the method has been applied to quantization of a complex scalar field.

The proposed modification of the standard procedures of canonical quantization is based on more consecutive following to the restrictions of basic discrete symmetries. The main new fact used in [1,2] consists in that the creation-annihilation operators of antiparticles, *defined* as charge-conjugate ones to operators of particles, *are not identical* with operators in momentum decomposition of fields which earlier intuitively were identified with the first ones.

The principal new results are:

1) the ground state of the *C*-symmetric systems does not contain a zero-point energy and a zero-point charge;

2) it is in accordance with a generalization of the uncertainty relations for systems with non-Hermitian canonical variables;

3) as propagators for quanta of such systems can be chosen the retarded (particles) and advanced (antiparticles) propagators.

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In the present paper, which is continuation of [2], in sections 1 and 5 the *C*-symmetric quantization of massive vector and spinor fields is considered. Further the new method is extended by including the restrictions following from the chiral symmetry of systems also. It is allowed to show in sections 2-3 lack of a zero-point energy for the neutral fields also - electromagnetic field and non-Abelian gauge fields quantized in the helicity basic.

1. *C*-symmetric quantization of a complex vector field

Complex vector field we can decompose on field modes similar to a complex scalar field, but with taking into account the polarization vector:

$$\begin{aligned} B_\mu(x) &= \sum_{\mathbf{k}\lambda}\left(a_{k\lambda}\varepsilon^{\lambda}_{\mu k}e^{-ikx} + \beta^*_{\lambda k}\varepsilon^{\lambda *}_{\mu k}e^{ikx}\right), \\ B^*_\mu(x) &= \sum_{\mathbf{k}\lambda}\left(a^*_{k\lambda}\varepsilon^{\lambda *}_{\mu k}e^{ikx} + \beta_{\lambda k}\varepsilon^{\lambda}_{\mu k}\, e^{-ikx}\right). \end{aligned} \tag{1}$$

Since the field equations for a free field are linear, in each frame of reference three states of polarization are quantized independently and are similar to three complex scalar fields. Therefore, reducing details, we will present only results.

At quantization the operators of field modes satisfy the commutators, nonzero of which look like:

$$\left[a_{k\lambda}, a^*_{k'\lambda'}\right] = \left[\beta_{k\lambda}, \beta^*_{k'\lambda'}\right] = \delta_{\lambda\lambda'}\delta^3(\mathbf{k}-\mathbf{k}'). \tag{2}$$

The Hamiltonian and charge operator then take a form:

$$\begin{aligned} H &= \sum_\lambda \int d^3k\, H_{k\lambda} = \sum_\lambda \int d^3k\, \omega_k \left(a^*_{k\lambda}a_{k\lambda} + \beta_{k\lambda}\beta^*_{k\lambda}\right), \\ Q &= \sum_\lambda \int d^3k\, Q_{k\lambda} = \sum_\lambda \int d^3k \left(a^*_{k\lambda}a_{k\lambda} - \beta_{k\lambda}\beta^*_{k\lambda}\right). \end{aligned} \tag{3}$$

By introducing the charge-conjugate operators for the field modes and using the *C*-symmetry requirements for observables of each polarization state separately, two products of auxiliary operators $\beta_{k\lambda}\beta^*_{k\lambda}$ and $\alpha_{k\lambda}\alpha^*_{k\lambda}$ we express through two products of the charge-conjugate operators:

$$\alpha_k\alpha^*_k = a^*_k a_k\,, \qquad \beta_k\beta^*_k = b^*_k b_k\,. \tag{4}$$

By substituting these identities into (3), for the Hamiltonian and the charge operator of the complex vector field we obtain the standard normal-ordered expressions without a zero-point energy and a zero-point charge:

$$\begin{aligned} H = H_c &= \sum_\lambda \int d^3k\, H_{k\lambda} = \sum_\lambda \int d^3k\, \omega_k \left(a^*_{k\lambda}a_{k\lambda} + b^*_{k\lambda}b_{k\lambda}\right), \\ Q = Q_c &= \sum_\lambda \int d^3k\, Q_{k\lambda} = \sum_\lambda \int d^3k \left(a^*_{k\lambda}a_{k\lambda} - b^*_{k\lambda}b_{k\lambda}\right). \end{aligned} \tag{5}$$

Let's notice that though this result is obtained in a fixed frame of reference, nevertheless, the fact of lack of the zero-point energy and the zero-point charge does not depend on a frame of reference. It is related by that the energy-momentum tensor of vacuum Λg_{ik} is proportional to a scalar constant and, therefore, if it vanishes in one a frame, then it vanishes in other frames also.

2. *C*-symmetric quantization of electromagnetic field

At quantization of electromagnetic field, in spite of the fact that photons are electrically neutral and the field is described by a real vector potential A_μ, nevertheless it is possible to use the properties of a discrete symmetry for the elimination of a zero-point energy of vacuum.

The photon field has an axial symmetry and in circular polarization two states with opposite helicities behave exactly as two states with opposite chiral charges. The Hamiltonian and a helicity operator $\Lambda = \mathbf{S}\cdot\mathbf{k}/|\mathbf{k}|$ (projection of spin to a momentum direction) are diagonal only at the circular polarization and, therefore, only this polarization corresponds to pure states of photons. As a result, the earlier developed *C*-symmetric quantization formalism is applicable almost completely to this case, but with replacement of a charge on a helicity, which in fact is a chiral charge of photon.

The helicity operator Λ^0 at the lack of an angular momentum is a time component of a divergence on a spin operator:

$$\Lambda^\mu = \partial_\nu J^{\mu\nu}. \tag{6}$$

Let's choose an axis x^3 of spatial coordinates along the direction of photon's momentum $\mathbf{k} = (0,0,k^3)$, i.e. we will choose a gauge where there remain only transverse physical components of the field A_1, A_2. Further, we combine these two transverse components to two circular components with helicities $\lambda = \pm 1$, which are mutually complex conjugate:

$$A(x) \equiv A_+(x) = \frac{1}{\sqrt{2}}\left(A_1 + iA_2\right), \quad A^*(x) \equiv A_-(x) = \frac{1}{\sqrt{2}}\left(A_1 - iA_2\right). \tag{7}$$

The field Lagrangian then takes a form:

$$\begin{aligned} L &= \frac{1}{2}\int d^3x\left[\left(\partial_\mu A_1\right)\left(\partial^\mu A_1\right) + \left(\partial_\mu A_2\right)\left(\partial^\mu A_2\right)\right] = \\ &= \int d^3x\left(\partial_\mu A^*\right)\left(\partial^\mu A\right) \end{aligned} \tag{8}$$

and leads to the Hamiltonian and the helicity operator:

$$H = \int d^3x\left[\pi\pi^* + \left(\nabla A^*\right)\cdot\left(\nabla A\right)\right], \tag{9}$$

$$\Lambda = i\int d^3x\left[\pi A - A^*\pi^*\right], \tag{10}$$

$$\pi(x) = \partial_t A^*, \quad \pi^*(x) = \partial_t A. \tag{11}$$

The field equations have the standard form

$$\partial_\mu\partial^\mu A = 0, \quad \partial_\mu\partial^\mu A^* = 0, \tag{12}$$

and equal time commutators look like:

$$\begin{aligned} &i\left[\pi(\mathbf{x},t), A(\mathbf{x}',t)\right] = \delta^3(\mathbf{x}-\mathbf{x}'), \\ &i\left[\pi^*(\mathbf{x},t), A^*(\mathbf{x}',t)\right] = \delta^3(\mathbf{x}-\mathbf{x}'). \end{aligned} \tag{13}$$

As we see, the photon field in the given gauge and the frame of reference formally is like a complex scalar field where the role of the charge operator plays the helicity operator. Therefore, the momentum decomposition of the field will be analogous to the decomposition of the complex scalar field [1]:

$$\begin{aligned} A(x)&=\sum_k\left(a_{k+}e^{-ikx}+\beta^*_{k-}e^{ikx}\right), \quad \pi^*(x)=-i\sum_k\omega_k\left(a_{k+}e^{-ikx}-\beta^*_{k-}e^{ikx}\right),\\ A^*(x)&=\sum_k\left(a^*_{k+}e^{ikx}+\beta_{k-}e^{-ikx}\right), \quad \pi(x)=i\sum_k\omega_k\left(a^*_{k+}e^{ikx}-\beta_{k-}e^{-ikx}\right). \end{aligned} \tag{14}$$

Here a_{k+}, a^*_{k+} are creation-annihilation operators of photons with the helicity $\lambda=+1$ and β_{k-}, β^*_{k-} are related to photons with the helicity $\lambda=-1$. Corresponding non-zero commutators for them look like:

$$\left[a_{\mathbf{k}+},a^*_{\mathbf{k}'+}\right]=\left[\beta_{k-},\beta^*_{k-}\right]=\delta^3\left(\mathbf{k}-\mathbf{k}'\right). \tag{15}$$

The Hamiltonian and the helicity operator thus take a form:

$$\begin{aligned} H&=\int d^3k\,H_k=\int d^3k\,\omega_k\left(a^*_{k+}a_{k+}+\beta_{k-}\beta^*_{k-}\right),\\ \Lambda&=\int d^3k\,\Lambda_k=\int d^3k\left(a^*_{k+}a_{k+}-\beta_{k-}\beta^*_{k-}\right). \end{aligned} \tag{16}$$

Here product of operators with the negative helicity $\beta_{k-}\beta^*_{k-}$ is not normally ordered. For transition to a normal-ordered operators we will use the charge-conjugation (*C*) symmetry transforming particles to antiparticles and backward. In our case it reduces to the changing a chiral charge, i.e. a helicity. There a right-polarised photon (particle) turns to a left-polarized photon (antiparticle) and backward.

Thus, the creation-annihilation operators subjected to C-conjugation turn to the creation-annihilation operators of photons of opposite helicity:

$$\begin{aligned} Ca_{k+}C^{-1}&=a_{k-}, \quad Ca^*_{k+}C^{-1}=a^*_{k-},\\ C\beta_{k-}C^{-1}&=\beta_{k+}, \quad C\beta^*_{k-}C^{-1}=\beta^*_{k+}. \end{aligned} \tag{17}$$

They appear in momentum decomposition of the *C*-transformed field functions:

$$\begin{aligned} A^c(x)&=\sum_k\left(a_{k-}e^{-ikx}+\beta^*_{k+}e^{ikx}\right)=CACP^{-1},\\ A^{c*}(x)&=\sum_k\left(a^*_{k-}e^{ikx}+\beta_{k+}e^{-ikx}\right)=CA^*C^{-1}. \end{aligned} \tag{18}$$

At the C-conjugation the Hamiltonian of each field mode is invariant, while the helicity operator changes a sign:

$$H^c_k\equiv C\,H_kC^{-1}=H_k, \quad \Lambda^c_k\equiv C\,\Lambda_kC^{-1}=-\Lambda_k, \tag{19}$$

where the expressions for the *C*-conjugate Hamiltonian and the helicity look like:

$$\begin{aligned} H^c&=\int d^3k\,H_{kP}=\int d^3k\,\omega_k\left(a^*_{k-}a_{k-}+\beta_{k+}\beta^*_{k+}\right),\\ \Lambda^c&=\int d^3k\,Q_{kP}=\int d^3k\left(a^*_{k-}a_{k-}-\beta_{k+}\beta^*_{k+}\right). \end{aligned} \tag{20}$$

Now the *C*-symmetry conditions (19) for the field modes we can write as:

$$H_k = \omega_k \left(a^*_{k+} a_{k+} + \beta_{k-} \beta^*_{k-} \right) = \omega_k \left(a^*_{k-} a_{k-} + \beta_{k+} \beta^*_{k+} \right) = H^c_k \,,$$
$$\Lambda_k = \left(a^*_{k+} a_{k+} - \beta_{k-} \beta^*_{k-} \right) = -\left(a^*_{k-} a_{k-} - \beta_{k+} \beta^*_{k+} \right) = -\Lambda^c_k \,, \tag{21}$$

and we see that they represent two coupling equations for four bilinear operator products:

$$a^*_{k+} a_{k+} + \beta_{k-} \beta^*_{k-} = a^*_{k-} a_{k-} + \beta_{k+} \beta^*_{k+} \,,$$
$$a^*_{k+} a_{k+} - \beta_{k-} \beta^*_{k-} = -a^*_{k-} a_{k-} + \beta_{k+} \beta^*_{k+} \,. \tag{22}$$

from which we find required identities:

$$\beta_{k+} \beta^*_{k+} = a^*_{k+} a_{k+} \,, \quad \beta_{k-} \beta^*_{k-} = a^*_{k-} a_{k-} \,. \tag{23}$$

By substituting these identities into (16), we obtain the expressions for the observables in the normal-ordered form:

$$H = \int d^3k \, H_k = \int d^3k \, \omega_k \left(a^*_{k+} a_{k+} + a^*_{k-} a_{k-} \right),$$
$$\Lambda = \int d^3k \, \Lambda_k = \int d^3k \left(a^*_{k+} a_{k+} - a^*_{k-} a_{k-} \right), \tag{24}$$

Thus, the observables are expressed through operators $a^*_{k\pm}, a_{k\pm}$ which create and annihilate the photons with helicities $\lambda = \pm 1$. Acting of these operators on the states is given by expressions:

$$a_{k+} \left| n_+ \right\rangle = \sqrt{n_+} \left| n_+ - 1 \right\rangle, \quad a^*_{k+} \left| n_+ \right\rangle = \sqrt{n_+ + 1} \left| n_+ + 1 \right\rangle,$$
$$a_{k-} \left| n_- \right\rangle = \sqrt{n_-} \left| n_- - 1 \right\rangle, \quad a^*_{k-} \left| n_- \right\rangle = \sqrt{n_- + 1} \left| n_- + 1 \right\rangle. \tag{25}$$

In particular, at $n_+ = 0$, $n_- = 0$ the annihilation operators define the vacuum:

$$a_{k+} \left| 0_+ \right\rangle = 0, \quad a_{k-} \left| 0_- \right\rangle = 0, \tag{26}$$

which any more does not contain the zero-point energy.

Thus, at quantization of the electromagnetic field in the specially chosen frame of reference and in the transverse gauge two field degrees of freedom are quantized by analogy to a complex scalar field. The photons with opposite helicities behave as a particle and an antiparticle.

As in the case of the complex vector field, the conclusion about lack of a zero-point energy of vacuum of the electromagnetic field, obtained in one a frame of reference and in one a gauge, is valid for all frames of reference and for all gauges because of invariance of properties of vacuum. Thus, at any moment the propagator of photons and an interaction vertex it is necessary to transform appropriately from helicity basic to that frame which is used, by taking into account the changing of the gauges also.

At first view the lack of a zero-point energy of photon field with a certain helicity looks paradoxical at the representation of fields as a set of harmonic oscillators for which have the zero-point energy. However, if to take into account that photons with a certain helicity are emitted by a rotating dipole, i.e. a rotator, the paradox disappears. Really, at first, the rotator spectrum in itself does not contain a zero-point energy and secondly, at circular polarization the vectors of electrical and magnetic field do not vibrate, but only rotate on a polarization plane. Oscillate only their projections to the fixed coordinate axes on this plane.

It basically differs from a case of emission of phonons in a crystal which happens at the harmonic vibrations of atoms. Here there is no *C*-symmetry, helicity states of phonons are dependent and transfer each other at a transformation of a frame of reference. The zero-point vibrations of atoms generate a fluctuating electromagnetic field which reveals as the Casimir effect.

3. *C*-symmetric quantization of non-Abelian gauge fields

Quantization of non-Abelian gauge fields A_μ^a with symmetry groups $SU(2)$, $SU(3)$,... differs on the electromagnetic field only with a number of vector fields and field's quanta are charged.

Nevertheless, if these fields are massless, the free Hamiltonian and a helicity operator of quanta possess axial symmetry and at the circular polarization are diagonal. Therefore, the above described method of *C*-symmetric quantization of the photon field is formally applicable in this case also, but with the account of restrictions following from the multi-component character of the fields and their self-interaction.

However, all additional contributions related by their difference from the photon field are proportional to a coupling constant g both in terms of their self-interaction and in the covariant derivatives. Therefore, if we will consider the non-Abelian gauge fields with a weak coupling constant $g^2 \ll 1$ these terms can be neglected and the field can be quantized as a set of independent photon degrees of freedom.

As it has been shown above, in this case the vacuum of the gauge field does not contain the zero-point energy following from a free Hamiltonian. Since the presence or lack of the zero-point energy is related to more fundamental physical reasons, than character of interaction of fields, at the further including of the interaction the vacuum energy, if appears, does not relate to the zero-point fluctuations. The contributions to the energy density of the physical vacuum will be proportional to a coupling constant and will not have any relation to the zero-point energy related to the procedure of quantization of the free fields.

Thus, at quantization of non-Abelian gauge fields in specially chosen frame, in transverse gauge and for weak coupling, when the field is similar to a photon field, the vacuum zero-point energy does not arise.

This conclusion about lack of the zero-point energy, obtained with these restrictions, is valid for all frames of reference and for all gauges because of invariance of properties of vacuum, since at values of coupling constants, when the perturbation theory is valid, the contributions of interactions concern to dynamical effects in the physical vacuum, not relating to the zero-point energy of the free Hamiltonian.

Certainly these conclusions concern to systems without a vacuum condensate (or at least to those regions where it is absent) and to situations where non-perturbative effects do not play a crucial role. The contributions of the condensates and topologically nontrivial solutions into the vacuum energy of fields will be considered in the further publications, but they also do not concern to the presence or lack of the zero-point energy of the free fields.

4. C-symmetric quantization of a spinor field

Let's consider a spinor field with a standard Lagrangian:

$$L = \int d^3x \bar{\psi} \left[i\gamma^\mu \left(\partial_\mu - eA_\mu \right) - m \right] \psi \ , \tag{27}$$

leading to the Hamiltonian and the charge operator (at interaction with a gauge field A_μ):

$$\begin{aligned} H &= \int d^3x \psi^+ \left(-i\boldsymbol{\gamma} \cdot \nabla + m \right) \psi \ , \\ Q &= \int d^3x \psi^+ \psi \, . \end{aligned} \tag{28}$$

The momentum decomposition of the spinor field looks like:

$$\begin{aligned} \psi(x) &= \sum_\alpha \int \frac{d^3p}{(2\pi)^{3/2}} \sqrt{\frac{m}{E_\mathbf{p}}} \left(b_{p\alpha} u_p^\alpha e^{-ipx} + \tilde{d}_{p\alpha}^+ v_p^\alpha e^{ipx} \right), \\ \psi^+(x) &= \sum_\alpha \int \frac{d^3p}{(2\pi)^{3/2}} \sqrt{\frac{m}{E_\mathbf{p}}} \left(b_{p\alpha}^+ u_p^{\alpha+} e^{ipx} + \tilde{d}_{p\alpha} v_p^{\alpha+} e^{-ipx} \right), \end{aligned} \tag{29}$$

where the normalization is $u_p^{\alpha+} u_p^{\alpha'} = v_p^{\alpha+} v_p^{\alpha'} = \delta^{\alpha\alpha'} E_p / m$ and $E_p = \sqrt{\mathbf{p}^2 + m^2}$.

Here important circumstance is that, as well as in the case of a scalar field [2], the operators $\tilde{d}_{p\alpha}^+$, $\tilde{d}_{p\alpha}$ are auxiliary ones and they cannot be directly interpreted as creation-annihilation operators of antiparticles. The last ones we will obtain further from the creation-annihilation operators of particles by their charge conjugation. The fact that thus defined true operators of antiparticles $d_{p\alpha}^+$, $d_{p\alpha}$ do not coincide with $\tilde{d}_{p\alpha}^+$, $\tilde{d}_{p\alpha}$ is a nontrivial consequence of the *C*-symmetry which will lead further to vanishing of a zero-point vacuum energy.

As it is known, the relativistic causality conditions require that spinor fields should be quantized by means of equal time *anti*commutators:

$$\begin{aligned} \left\{ \bar{\psi}(\mathbf{x}, t), \psi(\mathbf{x}', t) \right\} &= \delta^3 (\mathbf{x} - \mathbf{x}') I_{(4)} \, , \\ \left\{ \psi(\mathbf{x}, t), \psi(\mathbf{x}', t) \right\} &= \left\{ \bar{\psi}(\mathbf{x}, t), \bar{\psi}(\mathbf{x}', t) \right\} = 0 \, , \end{aligned} \tag{30}$$

from which follow non-zero anticommutators for field's modes:

$$\left\{ b_{p\alpha}, b_{p\alpha}^+ \right\} = \delta^3 (\mathbf{p} - \mathbf{p}') \delta_{\alpha\alpha'} \, , \quad \left\{ \tilde{d}_{p\alpha}, \tilde{d}_{p\alpha}^+ \right\} = \delta^3 (\mathbf{p} - \mathbf{p}') \delta_{\alpha\alpha'} \, . \tag{31}$$

The Hamiltonian and the charge operator of a free field are the sum of contribution of independent modes:

$$H=\sum_{\alpha}\int d^3p\,H_{p\alpha}=\sum_{\alpha}\int d^3p\,E_p\left(b^+_{p\alpha}b_{p\alpha}-\tilde{d}_{p\alpha}\tilde{d}^+_{p\alpha}\right),$$
$$Q=\sum_{\alpha}\int d^3p\,Q_{p\alpha}=\sum_{\alpha}\int d^3p\,\left(b^+_{p\alpha}b_{p\alpha}+\tilde{d}_{p\alpha}\tilde{d}^+_{p\alpha}\right). \tag{32}$$

By introducing the operators which are charge-conjugate to earlier introduced:

$$Cb_{p\alpha}C^{-1}=d_{p\alpha},\qquad Cb^+_{p\alpha}C^{-1}=d^+_{p\alpha},$$
$$C\tilde{d}_{p\alpha}C^{-1}=\tilde{b}_{p\alpha},\qquad C\tilde{d}^+_{p\alpha}C^{-1}=\tilde{b}^+_{p\alpha}, \tag{33}$$

the expressions for the charge-conjugate observables can be written in the form:

$$H_c=\sum_{\alpha}\int d^3p\,H_{p\alpha(c)}=\sum_{\alpha}\int d^3p\,E_p\left(d^+_{p\alpha}d_{p\alpha}-\tilde{b}_{p\alpha}\tilde{b}^+_{p\alpha}\right),$$
$$Q_c=\sum_{\alpha}\int d^3p\,Q_{p\alpha(c)}=\sum_{\alpha}\int d^3p\,\left(d^+_{p\alpha}d_{p\alpha}+\tilde{b}_{p\alpha}\tilde{b}^+_{p\alpha}\right), \tag{34}$$

where the *C*-symmetry requirements for each of independent field modes are:

$$H_{p\alpha(c)}\equiv CH_{p\alpha}C^{-1}=H_{p\alpha},\quad Q_{p\alpha(c)}\equiv CQ_{p\alpha}C^{-1}=-Q_{p\alpha}. \tag{35}$$

These requirements written in details:

$$H_{p\alpha}=E_p\left(b^+_{p\alpha}b_{p\alpha}-\tilde{d}_{p\alpha}\tilde{d}^+_{p\alpha}\right)=E_p\left(d^+_{p\alpha}d_{p\alpha}-\tilde{b}_{p\alpha}\tilde{b}^+_{p\alpha}\right)=H_{p\alpha(c)},$$
$$Q_{p\alpha}=\left(b^+_{p\alpha}b_{p\alpha}+\tilde{d}_{p\alpha}\tilde{d}^+_{p\alpha}\right)=-\left(d^+_{p\alpha}d_{p\alpha}+\tilde{b}_{p\alpha}\tilde{b}^+_{p\alpha}\right)=-Q_{p\alpha(c)}, \tag{36}$$

mean, in fact, following two *coupling equations* for four bilinear operator products:

$$b^+_{p\alpha}b_{p\alpha}-\tilde{d}_{p\alpha}\tilde{d}^+_{p\alpha}=d^+_{p\alpha}d_{p\alpha}-\tilde{b}_{p\alpha}\tilde{b}^+_{p\alpha},$$
$$b^+_{p\alpha}b_{p\alpha}+\tilde{d}_{p\alpha}\tilde{d}^+_{p\alpha}=-d^+_{p\alpha}d_{p\alpha}-\tilde{b}_{p\alpha}\tilde{b}^+_{p\alpha}. \tag{37}$$

By adding and subtracting these relations, two products of auxiliary operators $\tilde{d}_{p\alpha}\tilde{d}^+_{p\alpha}$ and $\tilde{b}_{p\alpha}\tilde{b}^+_{p\alpha}$ we can express through two products of the true creation-annihilation operators of particles and antiparticles:

$$\tilde{d}_{p\alpha}\tilde{d}^+_{p\alpha}=-d^+_{p\alpha}d_{p\alpha},\quad \tilde{b}_{p\alpha}\tilde{b}^+_{p\alpha}=-b^+_{p\alpha}b_{p\alpha}. \tag{38}$$

By substituted them into (32) and (34), we obtain the final expressions for H and Q, containing only the normal-ordered products of the creation-annihilation operators of mutually charge-conjugate quanta:

$$H=H_c=\sum_{\alpha}\int d^3p\,E_p\left(b^+_{p\alpha}b_{p\alpha}+d^+_{p\alpha}d_{p\alpha}\right),$$
$$Q=-Q_c=\sum_{\alpha}\int d^3p\,\left(b^+_{p\alpha}b_{p\alpha}-d^+_{p\alpha}d_{p\alpha}\right). \tag{39}$$

Let's notice that now the vacuum expectation both from a charge operator $\langle 0|Q|0\rangle=0$ and from the spatial components of the current disappear:

$$\int d^3x\langle 0|\mathbf{j}(x)|0\rangle=\int d^3x\langle 0|\bar{\psi}(x)\boldsymbol{\gamma}\psi(x)|0\rangle\sim$$
$$\sim\sum_{p\alpha}\langle 0|\tilde{d}_{p\alpha}\tilde{d}^+_{p\alpha}|0\rangle=\sum_{p\alpha}\langle 0|d^+_{p\alpha}d_{p\alpha}|0\rangle=0. \tag{40}$$

Thus, the vacuum of a spinor field with the Lagrangian (27) does not contain the zero-point energy and the zero-point charge and the observables are naturally normal-ordered due to the *C*-symmetry requirements.

In the case of a massless neutral spinor field, where there is a chiral symmetry and a conserved chiral charge, quantization of such field is similar to the above presented cases.

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