Theoretical Physics, Astrophysics and Cosmology
Vol. **1**, No 2, p. 28 – 41, v1, 29 September 2006
Online: TPAC: 2098-003 v2, 18 September 2012
ISSN 1819-9135; ISSN 1991-3915 (e)

DOI: 10.9751/TPAC.2098-003

Quantum field theory without divergences at a correct temporal integration

Zahid Zakir *

Abstract

Quantum mechanical trajectories for particles or fields, especially in path integrals, have not temporal differentials and the path integrals are defined only on a finite slice time lattice. Therefore, in QFT all integrals on energy should be taken before turning to the limit of very small times when they are finite. The renormalizable theories are invariant under the dilatations of the time lattice slice. It is a new space-time symmetry extending the Poincaré group and it has been discovered earlier in the momentum representation as the renormgroup. Thus, quantum mechanics requires turning to small times only after summation over all alternatives, i.e. energy integrations in loops, and this fact leads to natural regularization of loop integrals without additional hypotheses. These justify all effective methods of regularization as various realizations of the natural temporal regularization following from the fractal nature of paths. The covariant Planck time appears as a smallest time interval due to the gravitational proper time dilation and redshift of proper frequencies near the Planck energy source. Therefore, the standard QFT with such covariant and finite regularization becomes mathematically rigorous and physically consistent.

PACS: 11.10.Gh, 11.10.Hi, 11.15.Ha, 11.10.Ly, 12.20.-m
Key words: quantum mechanics, path integral, quantization, renormalization, lattice regularization, symmetries, vacuum energy, cosmological constant

Content

* *Centre for Theoretical Physics and Astrophyics*, *Tashkent, Uzbekistan;* zahidzakir@theor-phys.org

Introduction

In the standard quantum field theory (QFT) there are divergences in loop diagrams which have been removed by adding into the Lagrangians divergent counter terms. The necessity to operate with diverging terms in perturbation theory has been considered as an evidence of intrinsic incompleteness of the theory [1-4].

Simplest explanation of that may be that the terms of perturbation theory are finite, but only the standard formulation of QFT is incomplete and additional improvements are required. But many attempts to improve the formulations without changing the basic principles have not led to the required results, it was found that the difficulties with quantization also insuperable. As the result, during last decades most of attempts to avoid the divergences have been directed to changing the basic principles of QFT, particularly, the principle of locality [4], without any experimental evidence for such revision of the basic principles.

At such situation it will be reasonable to return again to more precise formulation of physical basis of the well working theory, by supposing that the problems with the divergences could be caused by shortcomings of the historically accepted formulations. This is relevant especially, since such previous attempts in practice were concentrated around small number of approaches, part of which has not been correctly understood and appropriately developed, others have not been justified and most of them have been reduced to technical recipes without concerning their physical meaning.

Particularly, the lattice regularization, very effective for numerical simulations of fields, exists in *two versions*. In the first one, introducing a *spatial* lattice, *time is continuous*, while the second one introduces a lattice for *space and time*. Thus, the discretization of space has been considered as more important than of time and the lattice regularization has been used as a technical trick only [3,5].

At the same time, *third* of possible versions of the lattice regularization which, as it will be shown below, has a direct physical meaning following from the principles of quantum mechanics, has not been developed, though has been often used at the definition of temporal evolutions. This version consists in the introduction of a finite *lattice on temporal coordinate* at continuous space, with the subsequent transition to very small slices. In this case the fields and events are defined locally in space, however, *temporal intervals* Δt between events have an element *of non-locality* as in all quantum systems.

As is well known, an open problem at the excluding of divergences is not the *renormalization* of regularized expressions, which is natural at the presence of interactions, but a physical reason for the procedure of *regularization*. In the present paper it will be argued that, even the preserving a *locality in space, there are fundamental physical reasons for regularization* and that all successful procedures of quantization implicitly contain these physical regularizations.

These physical reasons are following fundamental facts:

(1) *quantum-mechanical trajectories,* in a manifold of field functions also, *have not time derivatives* and have fractal properties under the temporal evolution [1],

(2) at the Planck distances l_{pl} there is a gravitational dilation of proper times with respect to a world time of distant observers.

The first property requires a small, but finite time intervals $\Delta t > 0$ between the causally related events, the second requires that the relativistically covariant Planck time τ_{pl} is in fact a minimal possible world time interval between the such events and that $\Delta t > \tau_{pl}$ (up to some numerical coefficient).

In all methods of quantization of fields the operations of time differentiation or integration have been introduced as finite differences or their sums and then turned to the limit of very small temporal intervals. However, the introduction of the temporal lattice with the finite $\Delta t > \tau_{pl}$ is enough for the convergence of the integrals in the loop diagrams. Therefore, in perturbation theory at the finite $\Delta t > \tau_{pl}$ the loop contributions remain finite and in the renormalizable theories will be small enough also.

It is sufficient that the discretization of time is the property of inertial frames of reference where all processes are describing. Therefore, the inertial frames with different time lattices Δt and $\Delta t'$ may be transformed to each other and this is *a new symmetry of such frames*, in addition to their rotations and translations. Such extended group of transformations of frames of reference, only having a physical meaning in the describing quantum systems, includes, in addition to the Poincaré group, also *a group of dilatations of the temporal lattice slice* $\Delta t \to \Delta t'$. The physical consequences of the new space-time symmetry have been discovered earlier in a hidden form at the description of fields in the momentum representation as a *renormgroup* symmetry [2-5].

In the first section it will be shown, that the turning to very small time intervals is possible only after summation over all alternatives, i.e. summation on energy and momentum, and that these operations cannot be interchanged due to the fractal nature of quantum-mechanical trajectories. In the second section it will be shown, that such changings in the formalism of QFT, taking into account the existence minimal interval $\Delta t > \tau_{pl}$, is enough for the natural covariant regularization of loop divergences. Such physical explanation of the renormalization methods will be considered in the section 3 where the new space-time symmetry also will be discussed.

1 A role of a temporal lattice in quantum mechanics

1.1 A fractal nature of quantum-mechanical trajectories

As it is known, for quantized systems (particles and fields) the canonical variables of classical mechanics have a physical meaning in the limits restricted by the uncertainty relations. However, the fact that the coordinates and momenta of quantum particles cannot be exactly measured simultaneously, of course, does not mean that the quantum particle have not trajectory in general. This means only that this is a new type of trajectory.

The classical trajectories are continuous and differentiable in time. It is well known that the quantum-mechanical trajectories also are continuous, but do not have time derivatives [1]. Notice that such property of trajectories is not something unusual even in the classical mechanics since a trajectory of a classical particle making Brownian motion also does not have time derivative and its coordinates and momenta also cannot be exactly measured simultaneously.

Thus, the fractal nature of the trajectories of the quantum particle may be demonstrated in an example of the mean-square value of velocity. In the time interval Δt it is inverse proportional to Δt:

$$\left\langle \left(\frac{x(t+\Delta t)-x(t)}{\Delta t} \right)^2 \right\rangle \sim \frac{1}{\Delta t} \tag{1}$$

and at the limit $\Delta t \to \varepsilon$ diverges as ε^{-1}. Here mean-square values of spatial paths thus are proportional to Δt, as it takes place for Brownian trajectories:

$$\left\langle \left[x(t+\Delta t)-x(t) \right]^2 \right\rangle \sim \Delta t. \tag{2}$$

Therefore, at the calculation of measurable values of observables containing the time derivatives of coordinate, it is necessary to perform all averagings at a finite Δt and only then to take the very small time limit $\Delta t \to \varepsilon$. The quantum mechanics is formulated so, that observables are finite at that limit.

1.2 Transition to small times only after summation over alternatives

In the formulation of quantum mechanics in terms of path integrals there should be summed the amplitudes of probabilities for all alternatives. Point particle's probability amplitudes $\psi(x,t)$ and $\psi(x',t')$ are related as:

$$\psi(x',t') = \int K\left(x',t';x,t\right)\psi(x,t)dx(t), \tag{3}$$

where the propagator $K\left(x',t';x,t\right)$ for a non-relativistic particle is given by the path integral [1]:

$$K\left(x',t';x,t\right) = \lim_{\Delta t \to \varepsilon} \prod_{i=1}^{N-1} \int \frac{dx(t_i)}{\sqrt{2\pi i \Delta t / m}} \exp\left\{ iL\left[\dot{x}(t_i), x(t_i)\right]\Delta t \right\}. \tag{4}$$

Here $N = (t'-t)/\Delta t$ and L - is the Lagrangian.

Here time is taken as a discrete one at the continuous spatial coordinates. It is important that in (4) the transition to $\Delta t \to \varepsilon$ should be performed *only* after all spatial integrations, i.e. *after the summation over all alternatives* at the given temporal lattice.

If we try to pass to very small times before the spatial integrations, the path integral diverges due to the factor $\Delta t^{-1/2} \to \varepsilon^{-1/2}$ containing in a measure of each spatial integral.

Notice, that in general the spatial integrals also should be approximated as sum of finite differences, however in coordinate representation it does not sufficient since one can formally integrate at any moment of time. Nevertheless, in phase space and in the Hamiltonian path integrals there is an effective restriction for a minimal phase volume by the value of the action quantum: $\Delta x \Delta p \geq h$.

Thus, in the path integral formulation the propagation of the quantum-mechanical particle occurs around classical trajectories in spacetime in each temporal slice Δt, but the quantum-mechanical trajectories are fractal and do not have time derivatives in the classical sense.

In the next paper [7] it will be shown that because of gravitational dilation of proper times at the Planck distances (with respect to the world times t of distant observers) there is a lower limit for world time intervals between causally related events of the order of Planck time $\Delta t > \tau_p$.

Thus, a combination of two fundamental physical phenomena – the fractal nature of quantum-mechanical trajectories and the gravitational dilation of proper times lead to an invariant and absolute regularization of the path integrals in quantum theory.

2 Quantization of fields on a temporal lattice

2.1 Fractal nature of trajectories in manifold of field functions

In quantum field theory the amplitudes of probability for the realization of the field configurations $\varphi(x,t)$ and $\varphi'(x',t')$ are $\Psi[\varphi(x,t)]$ and $\Psi[\varphi'(x',t')]$ correspondingly. These amplitudes of probability are related as:

$$\Psi\left[\varphi'(x',t')\right] = \int K\left[\varphi(x',t');\varphi(x,t)\right]\Psi\left[\varphi(x,t)\right]D\varphi(x,t), \tag{5}$$

Here the propagator $K\left[\varphi(x',t');\varphi(x,t)\right]$ is given by the functional integral [3]:

$$\begin{aligned} K\left[\varphi(\mathbf{x}',t');\varphi(\mathbf{x},t)\right] &= \lim_{\Delta t\to\tau_p}\prod_{i=1}^{N-1}\frac{1}{\sqrt{2\pi i\Delta t}}\int D\varphi\left(\mathbf{x}'',t_i\right)\times \\ &\times\exp\left\{i\left[\int L\left[\partial_t\varphi\left(\mathbf{x}'',t_i\right),\varphi\left(\mathbf{x}'',t_i\right)\right]d^3x\,\Delta t\;\right]\right\}, \end{aligned} \tag{6}$$

where $D\varphi(\mathbf{x}'',t_i)$ is the functional measure including the normalization factors of the spatial integrations.

As well as in the case of trajectories of particles, the passing to the limit $\Delta t \to \tau_p$ in (4) should be performed after all integrations on field configurations at the given temporal lattice. If we again try to pass to small times before the integrations on field configurations, than the propagator grows due to the factor $1/\Delta t^{1/2} \to 1/\tau_p^{1/2}$ contained in the measure of each integral on a field degree of freedom.

Thus, the propagation of field configurations at quantization occurs around trajectories in the manifold of field configurations and time, do not having time derivatives in the classical sense due to their fractal nature. Although time plays the special role in quantum theory, there do not arise the problems with the relativistic covariance at $\Delta t \to \tau_p$ due to the invariance of the Planck time τ_p.

2.2 Dependence of a bare Lagrangian on the temporal slices

The Lagrangian density of the bare scalar field φ_0 has the form:

$$L\left[\varphi_0, m_0, \lambda_0, \Lambda_0\right] = \frac{1}{2}\left(\partial_\mu \varphi_0\right)\left(\partial^\mu \varphi_0\right) - \frac{m_0^2}{2}\varphi_0^2 - \frac{\lambda_0}{4!}\varphi_0^4 - \Lambda_0, \quad (7)$$

where the bare field, the bare mass m_0, the interaction constant λ_0 and the vacuum energy density Λ_0 should be defined additionally. The bare fields and constants depend on the approximations which have been done at the formulation of the standard QFT formalism.

The *first approximation* is that, before calculation of matrix elements, the time derivatives in the expressions containing field variables should be taken in the form of finite differences and integrations - in the form of sum of finite differences. Therefore, the path integral in (6) contains only the contributions of quanta frequencies of which are smaller than some effective frequency $\omega_{\max} = \alpha\omega_0$, where $\omega_0 = 2\pi / \Delta t$, $\alpha < 1$.

However, the higher frequency contributions can not be simply ignored. Thus, the *second approximation*, which usually do not aware clearly, is that the contributions of quanta with frequency higher than $\omega_{\max}$, i.e. $\omega > \omega_{\max}$, are taken into account in the bare fields and the bare constants of the Lagrangian (7), which thus become depending on Δt, therefore, depend on $\omega_{\max}$ also. Since the physical Lagrangian $L\left[\varphi, m, \lambda, \Lambda\right]$ does not depend on Δt, then the quantization procedure supposes the separation two kind of contributions to it in the form:

$$L\left[\varphi, m, \lambda, \Lambda\right] = L\left[\varphi_0, m_0, \lambda_0, \Lambda_0; \Delta t\right] + L\left[\delta\varphi_0, \delta m_0, \delta\lambda_0, \delta\Lambda_0; \Delta t\right]. \quad (8)$$

Here the first term, the bare Lagrangian, expresses a contribution into the physical Lagrangian of depending on Δt quantum fluctuations with higher frequencies $\omega > \omega_{\max}$, while the second term – the Lagrangian of counter terms – contains the depending on Δt contributions of quanta with lower $\omega < \omega_{\max}$ frequencies.

Thus, since we shall start with the bare Lagrangian, it can be expressed through the physical Lagrangian as:

$$\begin{aligned} L\left[\varphi_0, m_0, \lambda_0, \Lambda_0; \Delta t\right] = \frac{1}{2}\left(\partial_\mu \varphi\right)\left(\partial^\mu \varphi\right) - \frac{m^2}{2}\varphi^2 - \frac{\lambda}{4!}\varphi^4 - \\ - \Lambda - L\left[\delta\varphi_0, \delta m_0, \delta\lambda_0, \delta\Lambda_0; \Delta t\right], \end{aligned} \quad (9)$$

where it depends on Δt in right hand side across the Lagrangian of counter terms only.

In the standard QFT, the action function with the bare Lagrangian has been defined *after* the continuous time limit. As the result, in the counter terms there had appeared the divergent contributions, although in renormalizable theories they were cancelled by introduction of counterterms.

In fact, in the renormalizable theories at the correct limit $\Delta t \to \tau_p$ there remain finite both counterterms and potentially growing loop contributions to the physical Hamiltonian.

2.3 Perturbation theory with a natural temporal regularization

At the summation over all alternatives restriction by the minimal (Planck) time $\Delta t \to \tau_p$ requires some modification of perturbation theory. In general, certainly, main features remain as before, but temporal integrals at field's localization points should be taken as sums of finite differences on time with $\Delta t > \tau_p$. At more rigorous approach introduction of spacetime boxes makes discrete the energies and momenta too, but in practical calculations it is not necessary doing that and we can maintain conveniences of the standard calculation technique.

Here we shall consider only some differences of the new treatment from the standard one. The decomposition of *S*-matrix as power series of the interaction constant has the form [2]:

$$S = 1 + \sum_{l \geq 1} \frac{1}{l!} \lim_{\Delta t \to \tau_p} \left[\sum_{n_1 = 1}^{N_1 - 1} \ldots \sum_{n_l = 1}^{N_l - 1} (\Delta t)^l \int d^3\mathbf{x}_1 \ldots d^3\mathbf{x}_l S(\mathbf{x}_1, n_1; \ldots; \mathbf{x}_l, n_l; \Delta t) \right], \quad (10)$$
$$S(\mathbf{x}_1, n_1; \ldots; \mathbf{x}_l, n_l; \Delta t) = i^l T\left[L(\mathbf{x}_1, n_1) \ldots L(\mathbf{x}_l, n_l) \right].$$

Here $l -$ is the degree the interaction constant, $t_l = n_l \Delta t$ - are moments of time when the quanta interact at the points $\mathbf{x}_l$. Thus, starting from the calculation of elements of *S*-matrix at the fixed Δt, only then it is necessary to pass to the smallest time limit $\Delta t \to \tau_{pl}$.

Notice, that the slices of time discretization describe the properties of global inertial frames of reference where the processes are described. There the times are measured by means of the set of identical standard clocks of the same slice Δt, all temporal slices Δt in integrals are identical and the time lattices for all diagrams on the same inertial frame are synchronized.

By using properties of the chronological products in (10) and by passing to the momentum representation, we obtain the conventional rules of the diagram techniques, but now only on the temporal lattice. There a number of energy integrations decreases due to the convergence of integrand functions at a finite Δt, when the integrals are convergent. Then, by separating from the integrals a part increasing at decreasing of Δt, we should consider them as the quantum fluctuation's contribution to the bare Lagrangian, which by definition should be cancelled by the counter terms. The parts which are finite at the smallest time limit are the contributions of quantum fluctuations leading to observable effects.

As the result, the integrals in each order of perturbation theory are finite. However, these exact and finite integrals are complicated and it is possible to replace them by more simplified standard ones by directly *cutting* at the Planck energy $\omega_k < \Lambda_{pl}$. Thus, the artificial regularization methods of the standard QFT become having clear physical meaning as approximations to the exact and converging integrals of this new form of perturbation theory based on the correct transition to smallest time intervals.

These changings of the integration rules in perturbation theory of local QFT lead also to corresponding modified rules of the diagram techniques. We will consider them in the next section for loop diagrams.

3 Finiteness of loop diagrams at correct transition to continuous time

3.1 Finiteness of one-loop diagrams

A one-loop contribution of electron's self-energy diagram to *S*-matrix is [2]:

$$S_{(2)} = -e^2 \lim_{\substack{\Delta t \to \tau_p \\ N,N' \to N_p}} \sum_{n'=1}^{N'-1} \sum_{n=1}^{N-1} (\Delta t)^2 \int d^3\mathbf{x}_2 d^3\mathbf{x}_1 : \bar{\psi}\left(\mathbf{x}_2, t_{2,n'}\right) \gamma_\mu \times$$
$$\times S^c\left(\mathbf{x}_2 - \mathbf{x}_1, t_{2,n'} - t_{1,n}\right) \gamma^\mu D_0^c\left(\mathbf{x}_2 - \mathbf{x}_1, t_{2,n'} - t_{1,n}\right) \psi\left(x_1, t_{1,n}\right) : . \quad (11)$$

Here the operators are normal-ordered due to the same reason as in the case of free fields [6].

After inserting the momentum decomposition of operators and propagators, the matrix element has the form:

$$S_{(2)}(p) \sim -e^2 \lim_{\substack{\Delta t \to \tau_p \\ N \to N_p}} \sum_{n=1-N}^{N-1} \Delta t \int \frac{d^4k dE'}{k^2 + i\varepsilon} \frac{\gamma_\mu \left(\gamma_0 E' - \gamma(\mathbf{p} - \mathbf{k}) + m\right) \gamma^\mu}{E'^2 - (\mathbf{p} - \mathbf{k})^2 - m^2 + i\varepsilon} e^{i(E' + k_0 - E) n \Delta t}, \quad (12)$$

where $E, \mathbf{p}$ - are energy and momentum of an external electron. In the standard QFT the temporal integral gives the delta-function $\delta\left[E' - (E - k_0)\right]$ which removes the integral on E', expressing it through $E - k_0$. As the result, the integral has the form:

$$I_1(p) = \int \frac{d^4k}{k^2 + i\varepsilon} \frac{\gamma_\mu \left(\hat{p} - \hat{k} + m\right) \gamma^\mu}{(p-k)^2 - m^2 + i\varepsilon}, \quad (13)$$

which contains an ultraviolet divergence.

In fact, as it was argued in previous sections, one must integrate over energy-momentum before time integration at a finite $\Delta t > \tau_{pl}$. But, then the energy integration converges because of presence of the exponents highly oscillating at large values of argument. Therefore, due to the convergence of the integrand function, the energy integrals become effectively cutted an the Planck energy $E_p < \Lambda_{pl}$.

For obtaining of a main part of contributions it is enough to consider that the cutting is sharp and happens exactly at the Planch energy $\Lambda_{pl} = 1.221 \times 10^{19} \mathrm{GeV}$. At more precise numerical calculation a difference from this main contribution is less than percent. Then the main logarithmic expression in one-loop contributions has a value (at $\alpha = /137.036$):

$$A = \frac{\alpha}{\pi} \ln \frac{\Lambda_{pl}^2}{m^2} = 0.239. \quad (14)$$

In the limit $q^2 \to 0$ the one-loop contribution to the polarization operator $\Pi_{(1)}$ and, accordingly, to $Z_{3(1)}$ and to the square of a bare charge of electron $\delta e^2 = e^2 - e_0^2$ then will be about 8%, which gives also the bare value of α_0:

$$\begin{aligned} &\Pi_{(1)} \approx \frac{1}{3}A = 0.0798 \approx 7.98\%, \\ &Z_{3(1)} = 1 - \Pi_{(1)} \approx 0.92, \\ &\alpha_{0(1)} = \alpha Z_3^{-1} \approx 1/126.1 \end{aligned} \tag{15}$$

The one-loop correction $\delta m = m - m_0$ to the bare mass of electron m_0 then will be about 18%:

$$\begin{aligned} &\delta m_{(1)} \approx \frac{3}{4}A \cdot m = 0.18\, m = 0.092\,\mathrm{MeV}, \\ &m_{0(1)} \approx 0.82m = 0.418\, MeV, \\ &Z_{1(1)}^{-1} = Z_{2(1)}^{-1} = 1 - \frac{3}{4}A \approx 0.82. \end{aligned} \tag{16}$$

Other loop diagrams can be calculated similarly and the time regularization in all cases is sufficient for obtaining of finite one-loop contributions to the scattering matrix.

3.2 Finiteness of multi-loop diagrams

At the standard proof of renormalizability of quantum electrodynamics in all orders of perturbation theory [2-5], it has been shown, that on mass shell $p_{(0)}^2 = m^2$ the subtractions of diverging terms in exact propagators and vertex functions of type are equivalent to the multiplicative renormalizations:

$$\tilde{S}_F = \frac{1}{Z_2} S'_F, \quad \tilde{D}_F = \frac{1}{Z_3} D'_F, \quad \tilde{\Gamma}'_\mu = Z_1 \Gamma_\mu. \tag{17}$$

Thus, it has been shown, that all divergences are contained only in these constants of renormalization $Z_1 = Z_2$, Z_3, and that these constants together with the counter terms in the Lagrangian lead to the converging perturbation theory.

Since one loop contributions are finite due to the natural temporal regularization with $\Delta t > \tau_p$, all new multi-loop contributions also are finite and in the renormalizable theories all such regularized terms can be represented as the finite constants of renormalization Z_i.

Thus, at the temporal regularization the proof of renormalizability of quantum electrodynamics (and other field theories) in all orders of perturbation theory simplifies and become physically meaningful and mathematically correct.

3.3 Vanishing of the physical vacuum energy in QFT

The last divergent loop contribution in the standard QFT was the vacuum loop diagrams. In the temporal regularization these diagrams also are finite, but can lead to the observing physical vacuum energy density Λ. At the same time, the physical value of Λ should be taken from the experimental data. The high accuracy measurements and corresponding exact calculations of anomalous magnetic moments of electron and muon without vacuum fields contributions show that, in fact, the experiments gives the vanishing vacuum energy $\Lambda = 0$ [6].

Thus, due to the vanishing of the vacuum energy, we can conclude that at least these contributions do not lead to the contradiction between particle physics and cosmology in the question about the smallness of the cosmological constant.

4 Some consequences of the temporal regularization

4.1 Standard regularizations as following from the temporal one

At the temporal regularization the convergence of momentum integrals is provided by the finiteness of a cutting energy – the Planck energy. For this reason, as in the standard QFT, the convergence formally also can be provided artificially by lowering the degree of growth of integrand $d^4k\,F(k)$.

The such lowering can be achieved by means of three groups of methods. In the first one we can lower a degree of growth of integrand function $F(k)$, in the second to reduce the degree of integration measure d^4k and in the third way to introduce a lattice for the coordinates catting the momentum integrals also.

In history of QFT the first way has been realized by introduction by hand of some functions leading to the convergence, or by simply cutting the integrals. The second way has been realized in the form of dimensional regularization. The third, lattice approach becomes a basic tool for studying of non-perturbative effects.

All these methods of regularization, in fact, are particular realizations of the temporal regularization, because of from the mathematical point of view it is not important how one achieves the convergence of energy integrals. A physical basis for it is the fact of finiteness of these integrals on energy at small times.

4.2 Renormgroup from the scaling at the temporal regularization

One of main properties of fractal structures is the similarity when a part of any fractal structure looks like as the whole structure and a part of this part also repeats the same. A trajectory of the particle at Brownian motion and quantum-mechanical trajectories in the path integrals have the same property of similarity [1]. Thus, at the last examples a role of the characteristic size played smoothed part of the trajectory passing by a particle during a time slice Δt. At decreasing the time slice:

$$\Delta t \to \Delta t / n, \tag{18}$$

there appear the same stochastic trajectories as before and so on up to very small temporal slices. The matrix elements for observables in quantum theory, beginning from some small value, become independent on further reducing of the time slices $\Delta t \to \Delta t'$.

On the other hand, in QFT the same symmetry of matrix elements has been discovered in the energy-momentum representation, named as *the renormgroup*, when in renormalizable theories the changing of a subtraction point of external momentum $p^2 = \mu^2$ in loop diagrams with corresponding redefinition of parameters of the Lagrangian does not change the observables [2-5].

Since at the temporal regularization there is a characteristic energy $\mu_{\Delta t} = 1/\Delta t$, it plays the role of a physically reasonable subtraction point in the diagrams. Thus, the fact, that temporal regularization has the scaling property and contains a natural subtraction point $\mu_{\Delta t}$, allows us to interpret the renormgroup as an empirically discovered form of the scaling property of the temporal regularization in the energy-momentum representation. As the result of the such interpretation, the renormgroup becomes a fundamental symmetry of QFT, following from its first principles, describing the property of similarity of the fractal quantum-mechanical trajectories for the field degrees of freedom.

Moreover, since the time discretization introduces a characteristic energy, only those theories have a physical meaning, which do not depend on that energy and they are renorm-invariant ones, or, renormalizable theories. Therefore, the property of renormalizability has the fundamental character and describes the invariance under a new, necessary extension of the group of relativistic transformations of frames of reference at the description of quantum systems which we consider in the next section.

4.3 A general space-time symmetry of the relativistic quantum theory

The standard QFT is symmetrical under the rotations and translations of the coordinate systems of inertial frames in flat space-time:

$$x'_\mu = \Lambda_{\mu\nu} x_\nu + a_\mu . \tag{19}$$

Here a symmetry group is the Poincaré group $ISO(1,3) = SO(1,3) \times T_4$, including 4-translations T_4 and the Lorentz group with generators $P_\mu = i\partial / \partial x^\mu$ and $J_{\mu\nu} = i(x_\mu \partial / \partial x^\nu - x_\nu \partial / \partial x^\mu)$ obeying the commutation relations:

$$\begin{aligned} \left[P_\mu, P_\nu\right] = 0,\ \left[P_\mu, J_{\nu\rho}\right] = i\left(g_{\mu\nu} P_\rho - g_{\mu\rho} P_\nu\right), \\ \left[J_{\mu\nu}, J_{\rho\sigma}\right] = i\left(g_{\nu\rho} J_{\mu\sigma} - g_{\mu\rho} J_{\nu\sigma} + g_{\mu\sigma} J_{\nu\rho} - g_{\nu\sigma} J_{\mu\rho}\right). \end{aligned} \tag{20}$$

Notice, that the physical coordinate axes in any inertial frame are constructed by using the standard rods and synchronized standard clocks.

Let the standard clocks are well known light clocks constructed as two parallel glasses with the periodically reflecting on them light signals. The half period of oscillations $\Delta t = \tau / 2$, when a light signal travels from the one glass to other, is a minimal temporal interval between the events which may be measured by these clocks. Therefore, the such constructed inertial frames have the minimal time slice Δt.

At the changing of a distance between parallel glasses this time slice also changes: $\Delta t \rightarrow \Delta t'$. This is trivial in classical physics a spacetime symmetry which, however, becomes non-trivial in quantum theory due to the fractal character of trajectories. Really, a particle's trajectory in a path integral at the time lattice with the slice Δt is approximated by a set of N short classical trajectories, while at the slices $\Delta t'$ it is approximated by another set of N' short classical trajectories. At values of Δt very small with respect to the characteristic times of the describing system, the path integral changes insufficiently at further reducing of the time slices. This region of Δt is a region of temporal scaling and there is a symmetry under the averaging of quantum fluctuations at more smaller and smaller time slices.

At any reducing of the time slice we take into account additional fluctuations describing by many new diagrams which was damped at the previous values of the time slice. In statistical physics the such symmetry has been discovered in form of the renormgroup when at averaging in larger and larger spatial blocks the parameters of the system become independent on the further averaging [3]. Therefore, the symmetry under the time slices transformations is the renormgroup symmetry of the quantum fluctuations of fields. In the energy-momentum representation this space-time symmetry appears as the usual renormgroup symmetry of the mass, charge and field functions renormalizations and here the analogy by the statistical physics becomes hidden.

Thus, a general space-time symmetry of the relativistic quantum theory is, in fact, a direct product of the Poincaré group and the renormgroup of the time lattice slices dilatation symmetry: $ISO(1,3)\times R_{\Delta t}$. The such generalized group of symmetry selects only a class of renormalizable theories as invariant ones under these generalized space-time transformations.

As an illustration let us consider a wave function for a particle which on the time lattices with slices Δt and $\Delta t'$ obeys the finite difference equations:

$$\begin{aligned} &\psi(x,t+\Delta t)=\left[1-iH_{\Delta t}\Delta t\right]\psi(x,t), \\ &\psi(x,t+\Delta t')=\left[1-iH_{\Delta t'}\Delta t'\right]\psi(x,t). \end{aligned} \tag{21}$$

Here $H_{\Delta t}$ - is the Hamiltonian, depending on Δt across the momenta. For the variation of the wave function we have:

$$\begin{aligned} &\delta\psi(x,t)=\psi(x,t+\Delta t)-\psi(x,t+\Delta t')= \\ &=\left[1-iH_{\Delta t}\delta\left(\Delta t\right)-i\delta H\,\Delta t\right]\psi(x,t). \end{aligned} \tag{22}$$

Properties of the system do not depend on the dilatations of time slices when $\delta H\approx 0$ and this is the renormgroup symmetry region for the system.

Thus, the time lattice renormgroup extension of the Poincaré group is similar to the Weyl extension including dilatations of d-dimensional coordinates:

$$x_{\mu}^{'}=\lambda x_{\mu}. \tag{23}$$

The generator of dilatations:

$$D=-i\left(d+x_{\mu}\frac{\partial}{\partial x_{\mu}}\right) \tag{24}$$

obeys the commutation relations:

$$\left[D,D\right]=0,\ \left[P_{\mu},D\right]=iP_{\mu},\ \left[J_{\mu\nu},D\right]=0\ . \tag{25}$$

Really, in the loop energy integrations the renormgroup region corresponds to the such values of energies which sufficiently exceed all external energies and masses of particles in the loop. In that case, it is possible to consider particles as massless ones and their motion as a light cone motion. Then the appearing of a symmetry concerning dilatations of coordinates becomes clear and natural.

But, nevertheless, renormgroup transformations of the time lattice, in fact, are opposite to dilatations of time coordinate. Really, let there are two inertial frames and let clocks on the first frame have the resolution Δt and on the second one: $\Delta t'=\Delta t/2.$ If an event has occurred at the moment of time $t=n\Delta t$ on clocks of the first frame, the same event will occur at the moment $t'=n'\Delta t'=t$ on the second frame and thus: $n'=2n$. So, the dilatations of time slices on the lattice:

$$\Delta t'=\lambda\,\Delta t. \tag{26}$$

remain unchanged the time coordinates of events and therefore:

$$t'=t,\quad n\Delta t=n'\Delta t'. \tag{27}$$

The generator of these transformations is similar to the generator of dilatations and has the form:

$$\tilde{D}=-i\left(1+\Delta t\frac{\partial}{\partial\left(\Delta t\right)}\right). \tag{28}$$

Since the time coordinate is represented now in the form: $t=n\Delta t$, then $\tilde{D}$ satisfies to the commutation relations as for the dilatations:

$$\left[\tilde{D},\tilde{D}\right]=0,\ \left[P_0,\tilde{D}\right]=iP_0,\ \left[J_{\mu\nu},\tilde{D}\right]=0\ . \tag{29}$$

Certainly, it is possible to consider a general case of a lattice for all coordinates, but the lattice on the spatial coordinates is a technical tool only up to limit $\Delta x\to l_{pl}$, in quantum theory a lattice on temporal coordinate has the special physical meaning and is inevitable at any scale.

More detailed discussion of properties of the such extension of the group of space-time symmetries will be presented in forthcoming publications.

Conclusion

Thus, as it is required by the physical nature of quantum systems and contained in the procedures of quantization, there is a natural temporal regularization of the integrals on energy with the subsequent continuous time limit $\Delta t\to 0$. This fact allows one to consider procedures of regularization in various methods of renormalization as the consequences of the temporal regularization.

Moreover, the counter terms in Lagrangians also depend on the time slices, they are finite at a finite Δt and then they cancel by similar contributions from loop diagrams. Therefore, QFT does not contain infinite renormalization constants. Loop contributions at the invariant gravitational regularisation due to the redshift of frequencies at the Planck energy are small enough, so perturbation theory appears as mathematically correct and physically consistent one.

Thus, by the solution of the problem of divergences, the long history of construction of QFT as a fundamental theory without internal contradictions is mainly finished. In future researches of new objects and application to new areas it is possible to be sure that deviations from predictions of QFT will be not due to the limitations of its principles, but only due to inefficiency of methods of calculation and the necessity of new models of matter and space-time structure.

References

1. Feynman R.P., Hibbs A.R. (1965) Quant. Mech. And Path Int., McGraw-Hill BC
2. Bogoliubov N.N., Shirkov D.V. (1980) Introd. Theory of Quant.Fields., Wiley.
3. Slavnov A.A., Faddeev L.D. (1988) Introd. Quant. Th. Gauge Fields., N., M.
4. Weinberg S. (1995-2000) The Quant. Theory of Fields. Vol. 1-3, CUP.
5. Wilson K.G., Kogut J. (1974) Phys. Rep., 12C, 2, p.75.
6. Zakir Z. (2006) Theor. Phys., Astroph. & Cosmol., **1**, 1, 11.
7. Zakir Z. (2006) Theor. Phys., Astroph. & Cosmol., **1**, 3, 42.